Eidgenössische Technische Hochschule Zürich
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Gravity Surveying

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Introduction

Gravity surveying…

Investigation on the basis of relative variations in the Earth´gravitational field arising from difference of density between subsurface rocks

Application

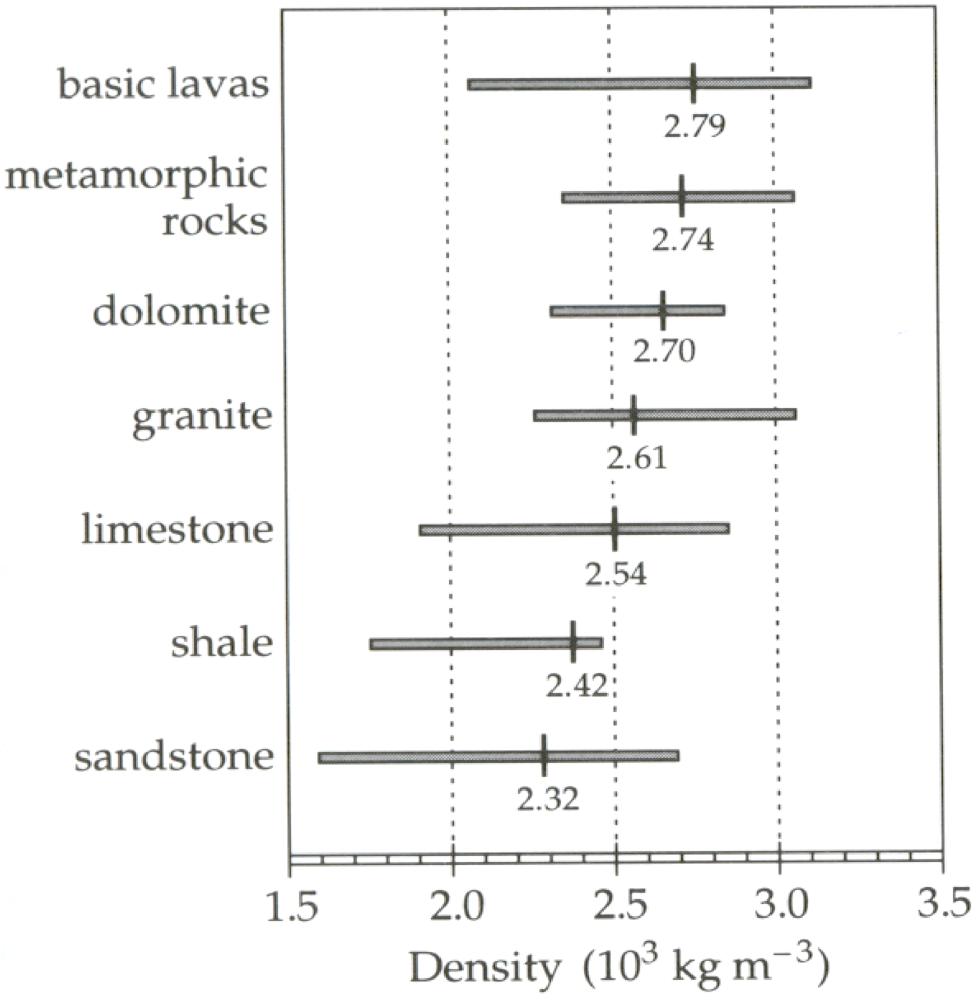
- Exploration of fossil fuels (oil, gas, coal)
- Exploration of bulk mineral deposit (mineral, sand, gravel)
- Exploration of underground water supplies
- Engineering/construction site investigation
- Cavity detection
- Glaciology
- Regional and global tectonics
- Geology, volcanology
- Shape of the Earth, isostasy
- Army

Structure of the lecture

1. Density of rocks
2. Equations in gravity surveying
3. Gravity of the Earth
4. Measurement of gravity and interpretation
5. Microgravity: a case history
6. Conclusions

1. Density of rocks

Rock density



Rock density depends mainly on…

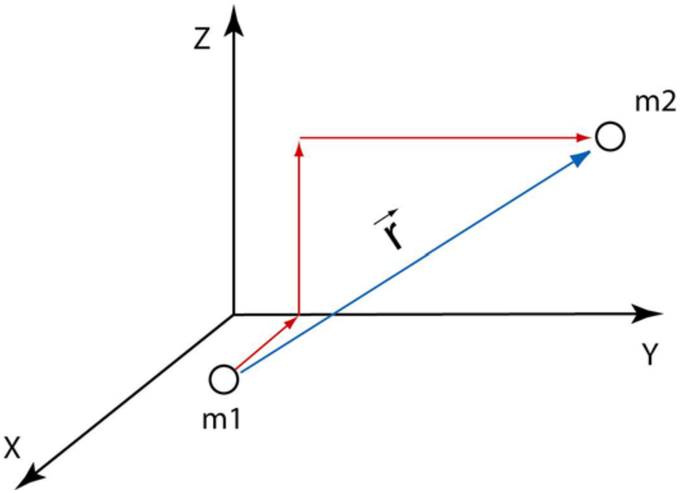
- Mineral composition
- Porosity (compaction, cementation)

Lab or field determination of density is useful for anomaly interpretation and data reduction

2. Equations in gravity surveying

First Newton´s Law

Newton´s Law of Gravitation



$$\vec{F} = -\frac{G\, m_1\, m_2}{r^2}\,\vec{r}$$

$$|r| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Gravitational constant $G = 6.67 \times 10^{-11} \quad \mathrm{m^3 kg^{-1} s^{-2}}$

Second Newton´s Law

$$\vec{F} = m\,\vec{a}$$

$$\vec{a} = -\frac{G\,M}{R^2}\,\vec{r} = \vec{g}_N$$

$$g_N \cong 9.81 \quad \mathrm{m/s^2}$$

g_N: gravitational acceleration or „gravity“

for a spherical, non-rotating, homogeneous Earth, g_N is everywhere the same

$M = 5.977 \times 10^{24} \quad \mathrm{kg}$ mass of a homogeneous Earth

$R = 6371 \quad \mathrm{km}$ mean radius of Earth

Units of gravity

- 1 gal = 10^{-2} m/s^2
- 1 mgal = 10^{-3} gal = 10^{-5} m/s^2
- 1 μgal = 10^{-6} gal = 10^{-8} m/s^2 (precision of a gravimeter for geotechnical surveys)

- Gravity Unit: 10 gu = 1 mgal

- Mean gravity around the Earth: 9.81 m/s^2 or 981000 mgal

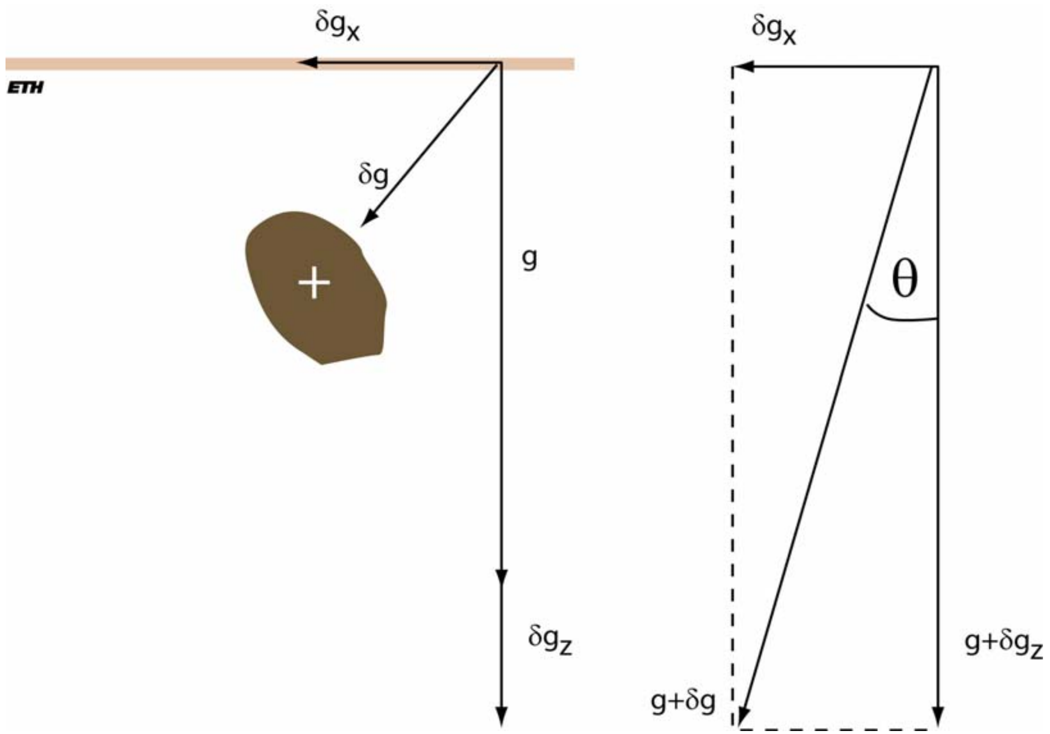
Keep in mind…

…that in environmental geophysics,
we are working with values about…

0.01-0.001 mgal ≈ 10^{-8} - 10^{-9} g_N !!!

Measurement component

The measured perturbations in gravity effectively correspond to the vertical component of the attraction of the causative body

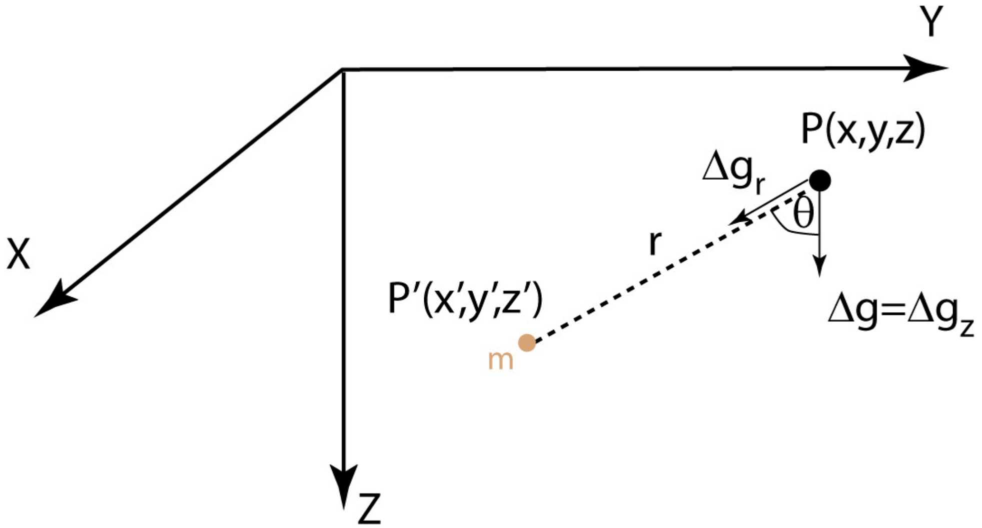


we can show that θ is usually insignifiant since $\delta g_z << g$

Therefore…

$$\delta g \approx \delta g_z$$

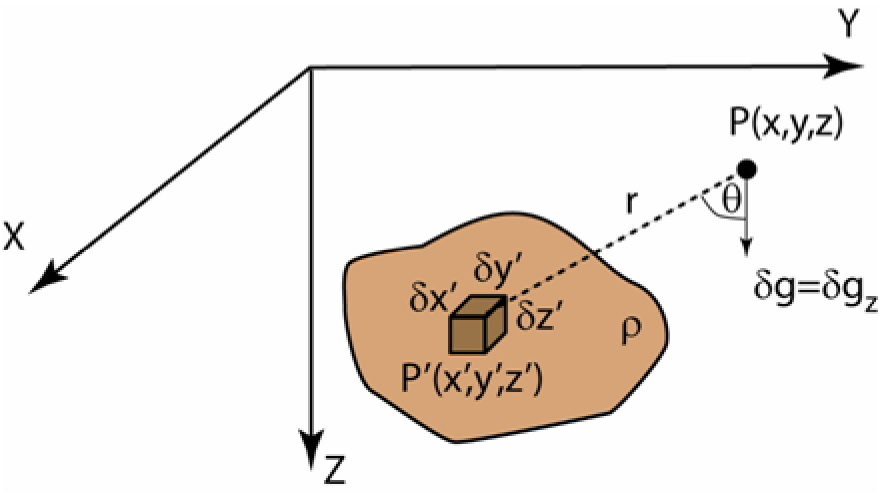
Grav. anomaly: point mass



$$\Delta g_r = \frac{Gm}{r^2} \quad \text{from Newton's Law}$$

$$\Delta g = \Delta g_z = \frac{Gm}{r^2}\cos\theta = \frac{Gm(z' - z)}{r^3}$$

Grav. anomaly: irregular shape



$$\Delta g = \frac{Gm(z' - z)}{r^3}$$

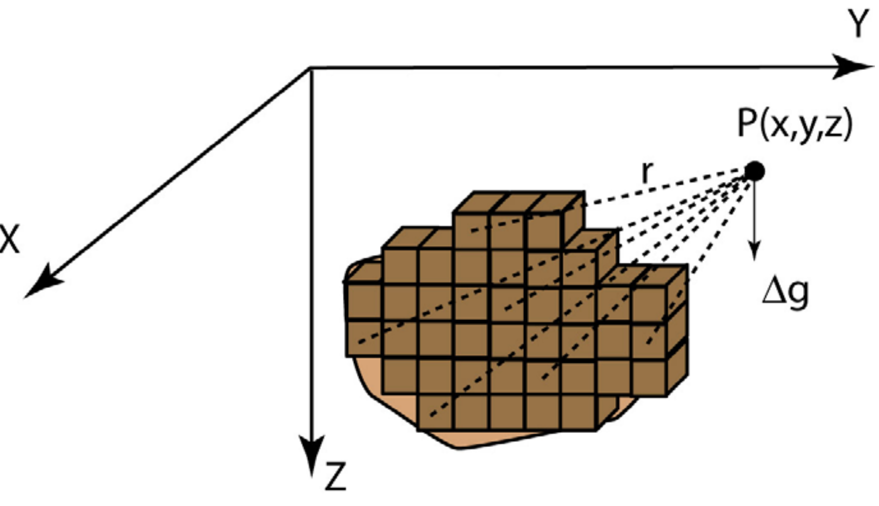
for $\delta m = \rho\, \delta x' \delta y' \delta z'$ we derive:

$$\delta g = \frac{G\rho(z' - z)}{r^3} \delta x' \delta y' \delta z'$$

with ρ the density (g/cm^3)

$$r = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2}$$

Grav. anomaly: irregular shape



for the whole body:

$$\Delta g = \sum\sum\sum \frac{G\rho(z' - z)}{r^3}\delta x'\delta y'\delta z'$$

if $\delta x', \delta y'$ and $\delta z'$ approach zero:

$$\Delta g = \iiint \frac{G\rho(z' - z)}{r^3} dx'dy'dz'$$

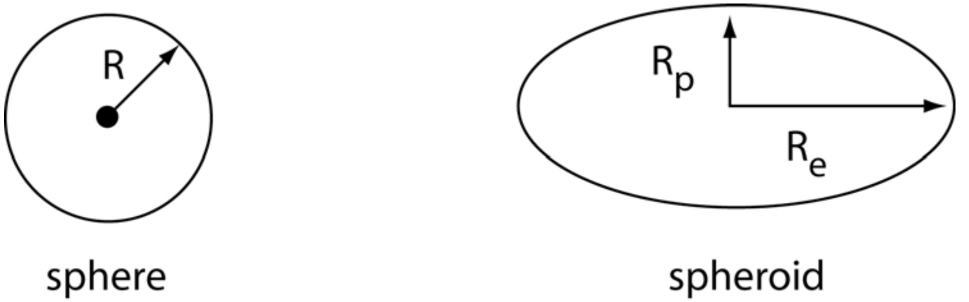
Conclusion: the gravitational anomaly can be efficiently computed! The direct problem in gravity is straightforward: Δg is found by summing the effects of all elements which make up the body

3. Gravity of the Earth

Shape of the Earth: spheroid

- Spherical Earth with R=6371 km is an approximation!

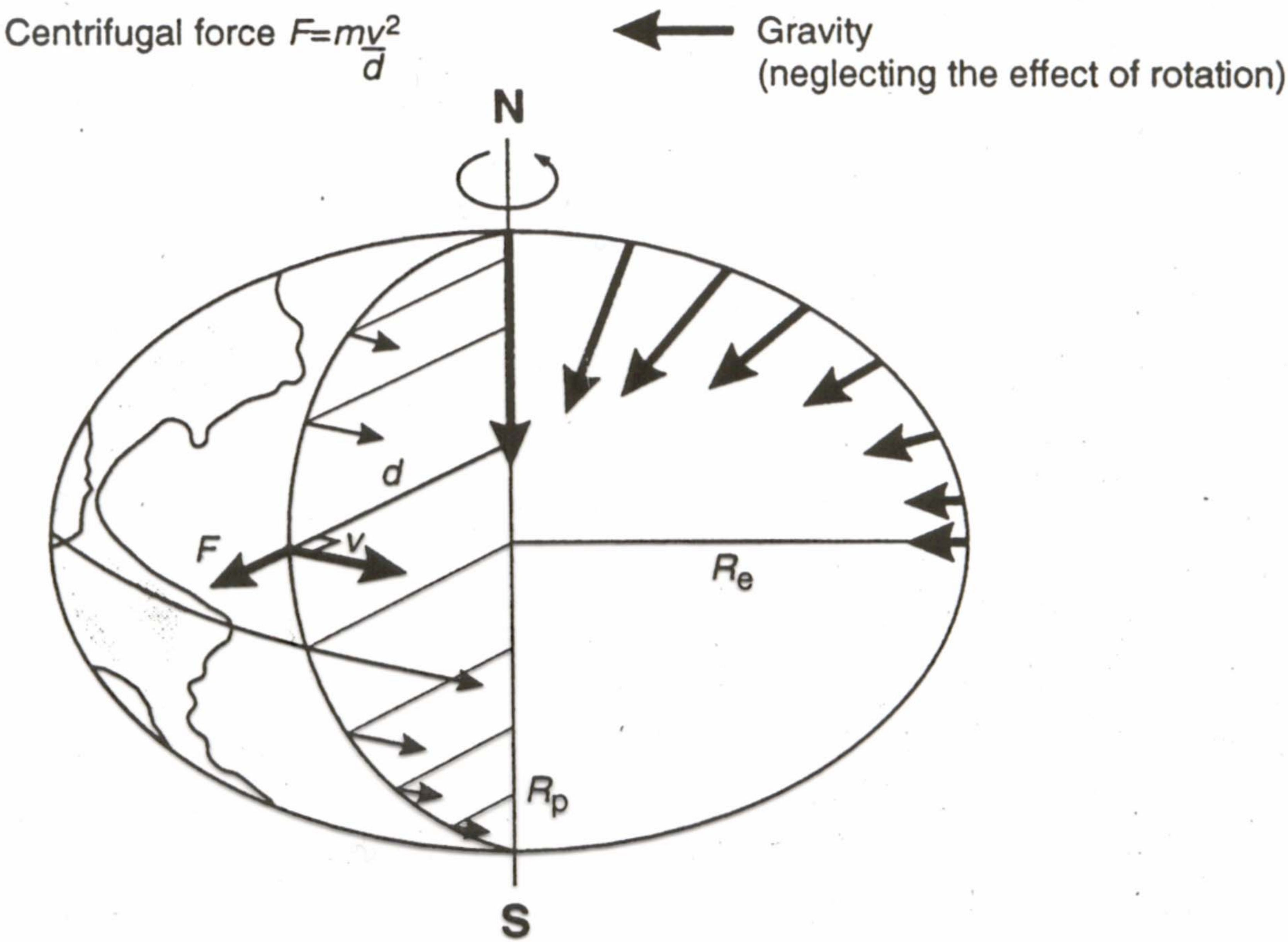
- Rotation creates an ellipsoid or a spheroid

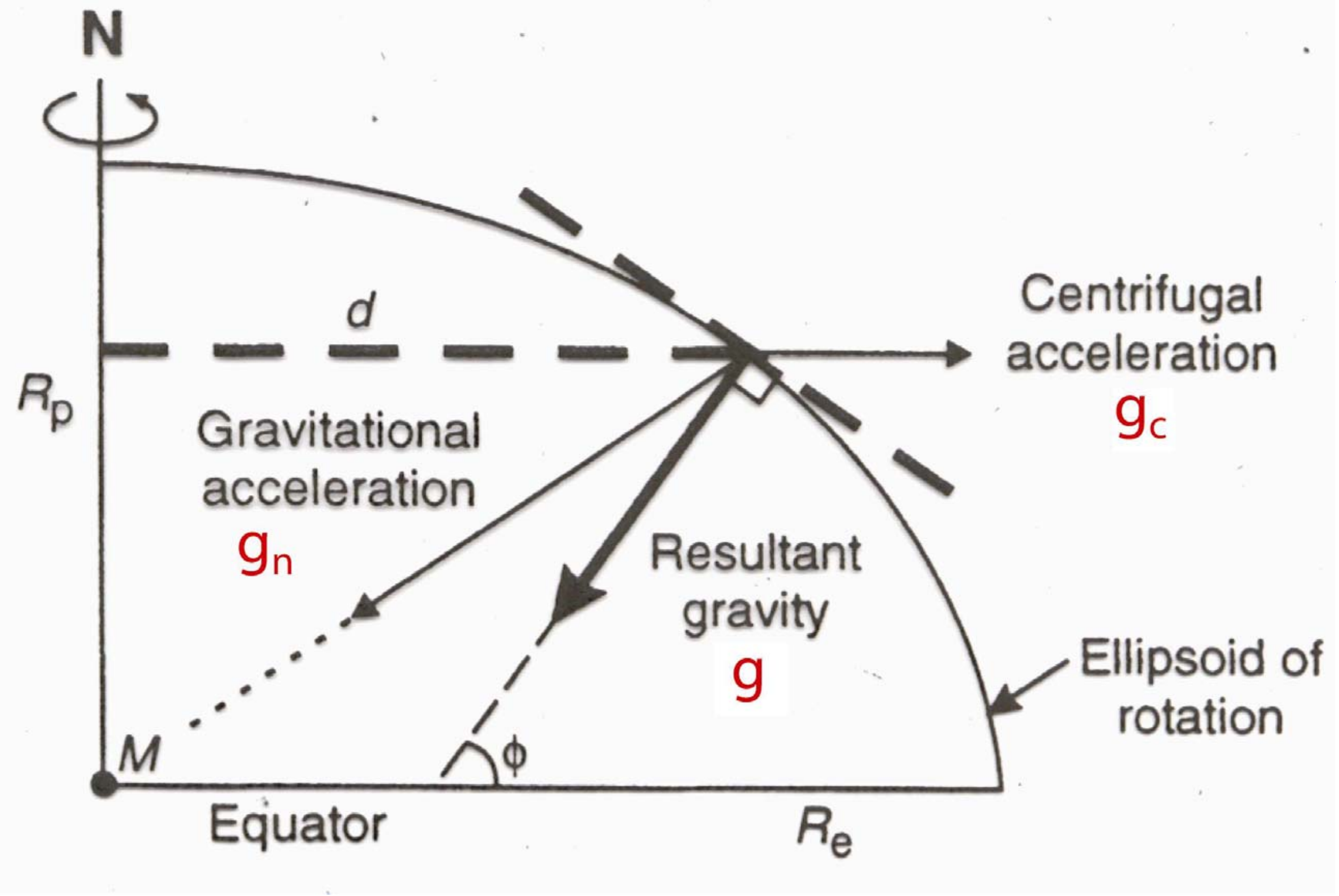


$$\frac{R_e - R_p}{R_e} = \frac{1}{298.247}$$

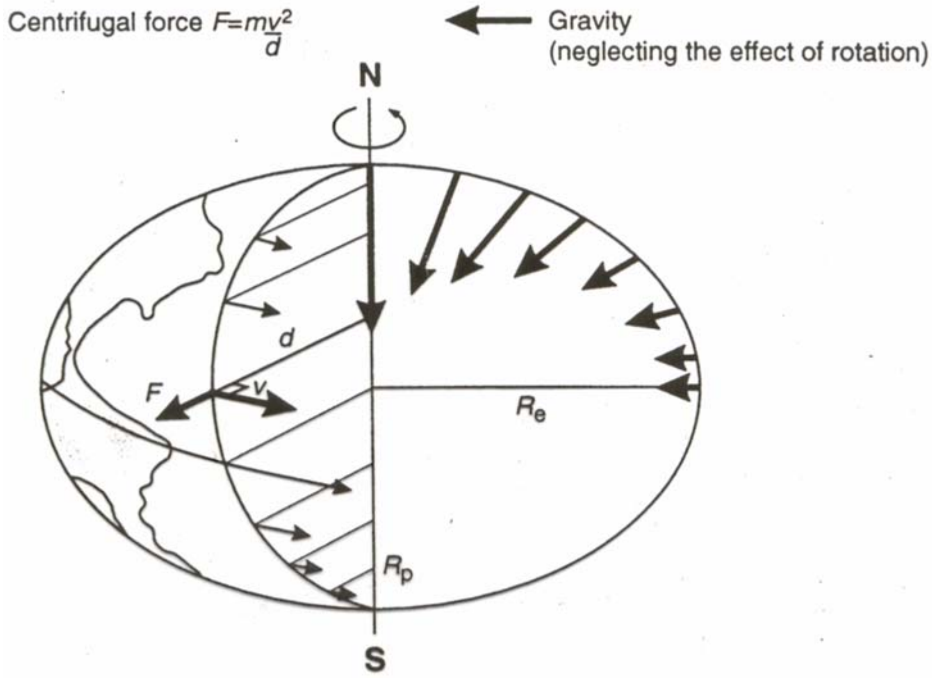
Deviation from a spherical model:

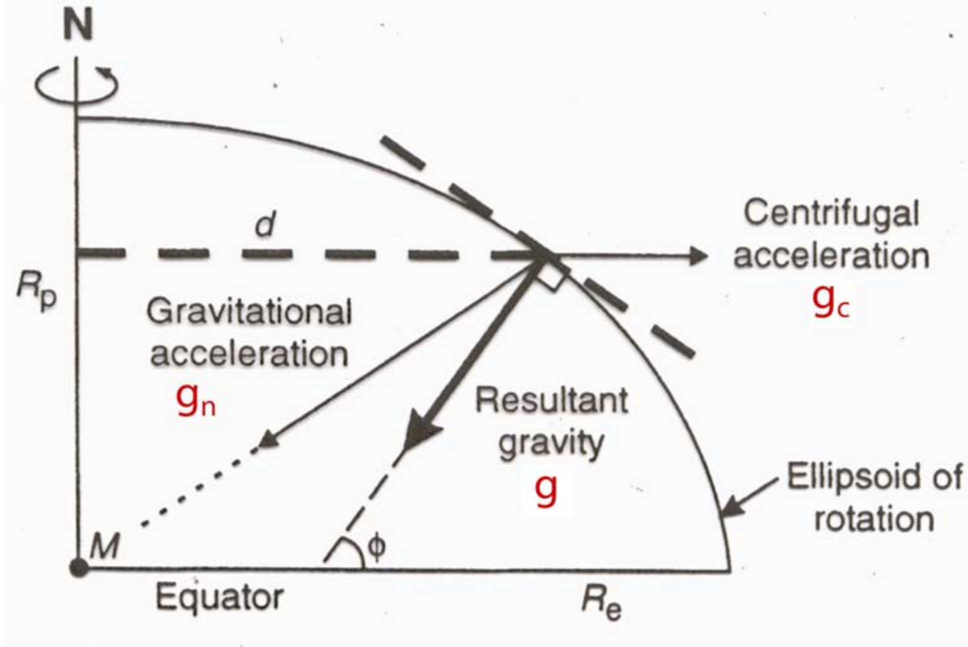
$$R_e - R = 7.2 \quad \text{km}$$
$$R - R_p = 14.3 \quad \text{km}$$


Centrifugal force F=mv²/d
Gravity
(neglecting the effect of rotation)
N
S
d
F
v
Re
Rp



The Earth´s ellipsoidal shape, rotation, irregular surface relief and internal mass distribution cause gravity to vary over it´s surface





$$g = g_n + g_C = G\left(M/R^2 - \omega^2 R\cos\phi\right)$$

- From the equator to the pole: g_n increases, g_c decreases
- Total amplitude in the value of g: 5.2 gal

Reference spheroid

- The reference spheroid is an oblate ellipsoid that approximates the mean sea-level surface (geoid) with the land above removed

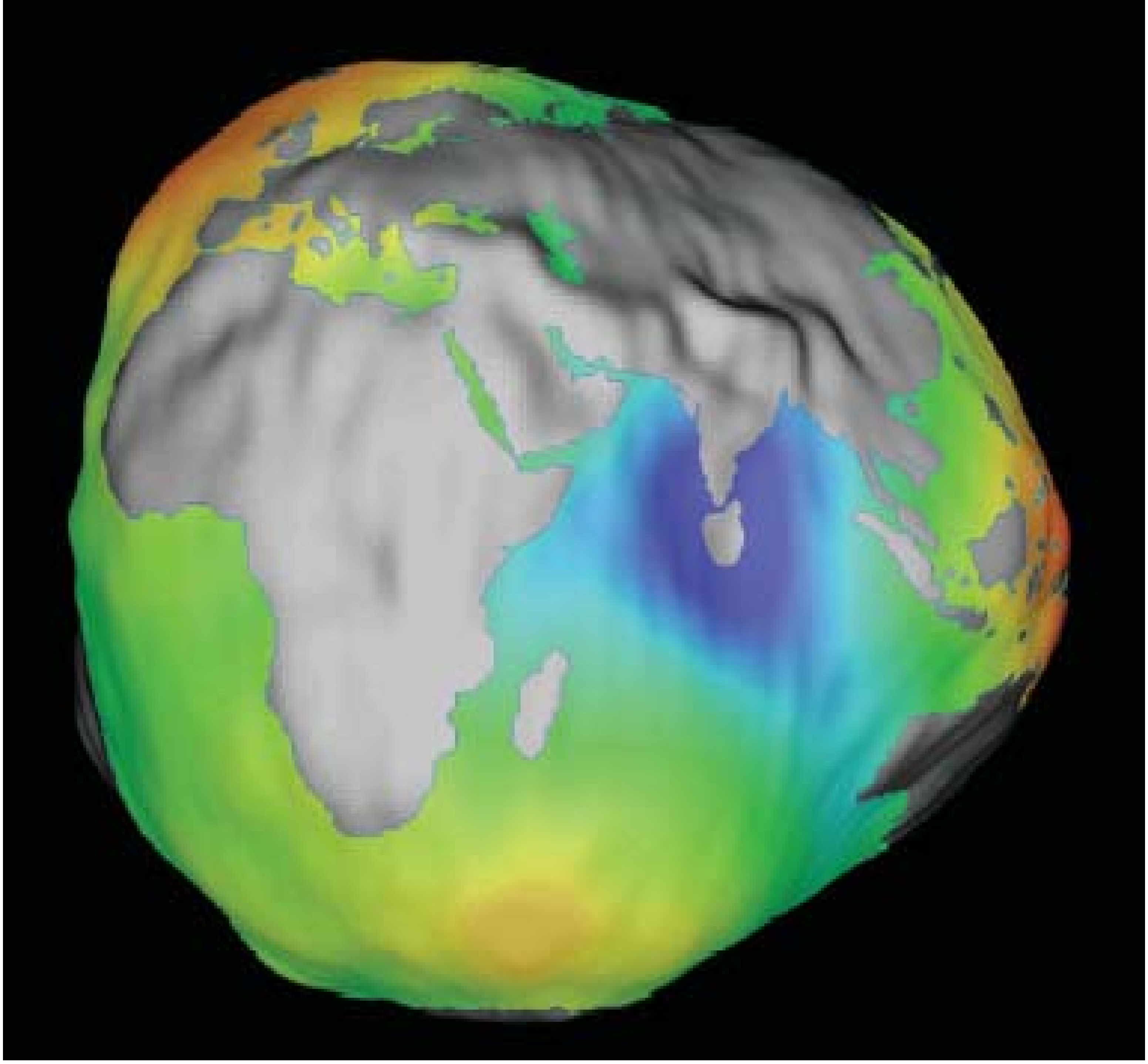
- The reference spheroid is defined in the Gravity Formula 1967 and is the model used in gravimetry

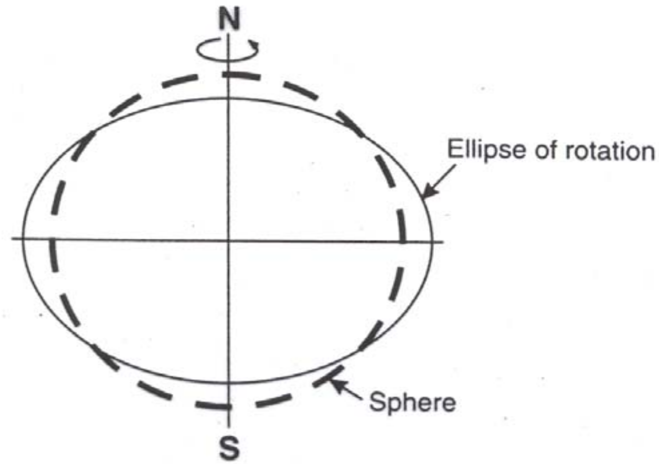
- Because of lateral density variations, the geoid and reference spheroid do not coincide

Shape of the Earth: geoid

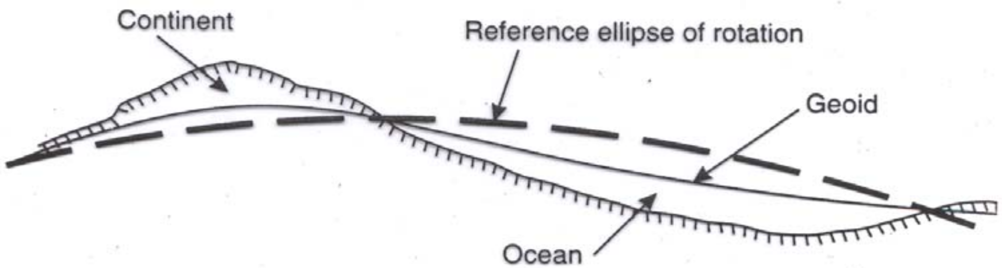
- It is the sea level surface (equipotential surface)

- The geoid is everywhere perpendicular to the plumb line

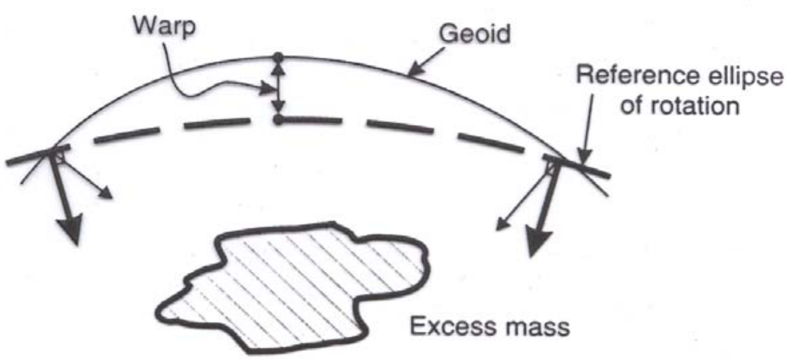



N
Ellipse of rotation
Sphere
S

(A)

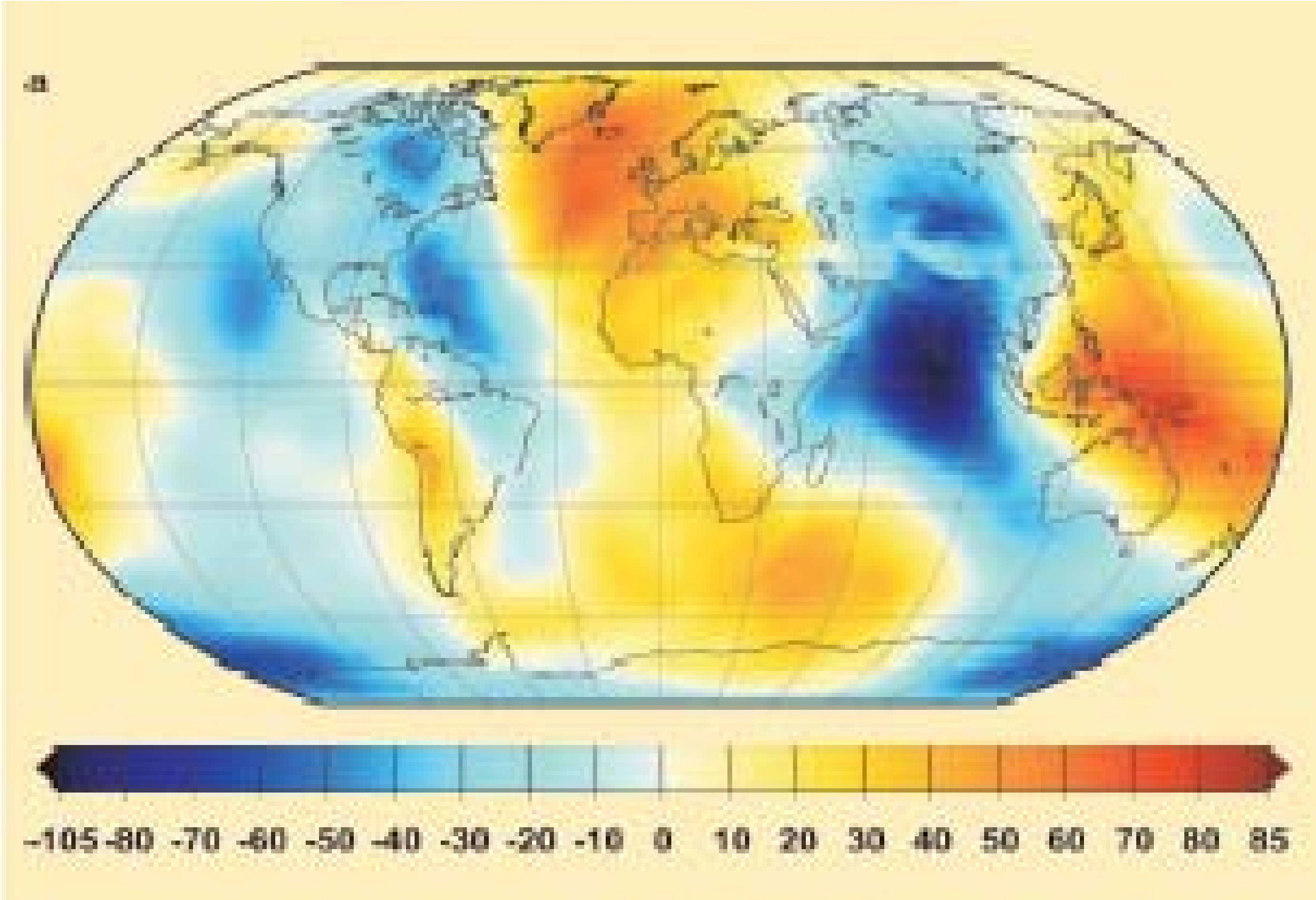

Continent
Reference ellipse of rotation
Geoid
Ocean

(B)


Warp
Geoid
Reference ellipse
of rotation
Excess mass

Spheroid versus geoid

Geoid and spheroid usually do not coincide (India -105m, New Guinea +73 m)



4. Measurement of gravity and interpretation

Measurement of gravity

Absolute measurements

- Large pendulums

$$T = 2\pi \sqrt{L/g}$$

- Falling body techniques

$$z = \frac{1}{2} g\, t^2$$

For a precision of 1 mgal

Distance for measurement 1 to 2 m

z known at 0.5 µm

t known at 10^{-8} s

Relative measurements

- Gravimeters
- Use spring techniques
- Precision: 0.01 to 0.001 mgal

Relative measurements are used since absolute gravity determination is complex and long!

Gravimeters

LaCoste-Romberg mod. G

Scintrex CG-5

Source: P. Radogna, University of Lausanne

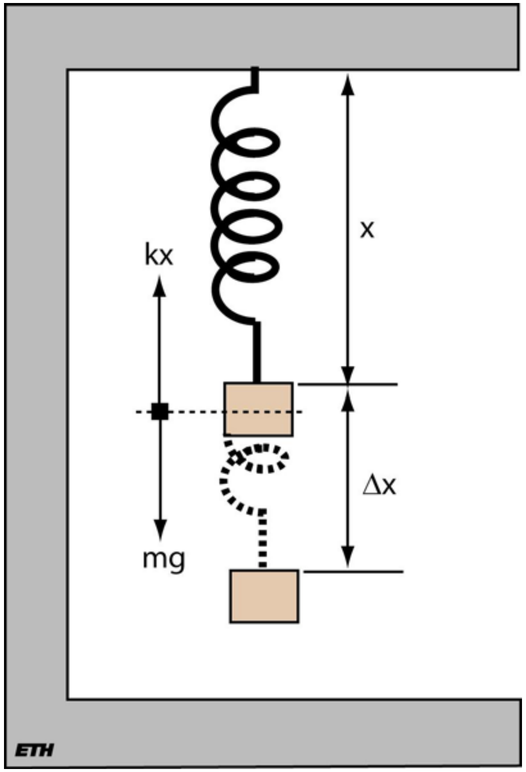
Stable gravimeters

$$\Delta g = \frac{k}{m} \Delta x$$ Hook´s Law

$$g = \frac{4\pi^2}{T^2} \Delta x \quad \text{with} \quad T = 2\pi \sqrt{\frac{m}{k}}$$

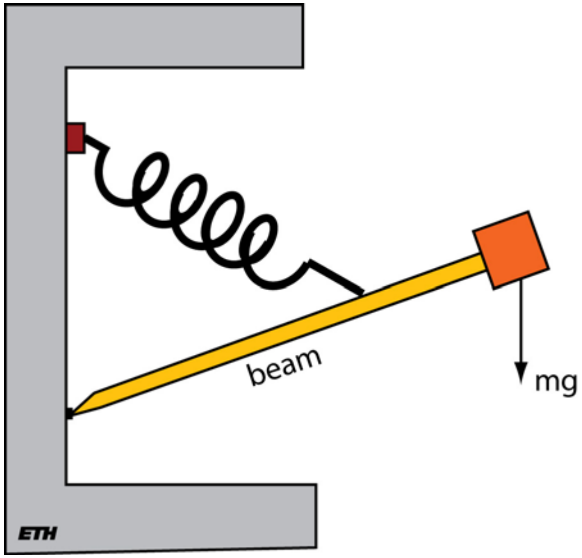
For one period

k is the elastic spring constant



Problem: low sensitivity since the spring serves to both support the mass and to measure the data. So this technique is no longer used…

LaCoste-Romberg gravimeter



This meter consists in a hinged beam, carrying a mass, supported by a spring attached immediately above the hinge.

A „zero-lenght“ spring can be used, where the tension in the spring is proportional to the actual lenght of the spring.

- More precise than stable gravimeters (better than 0.01 mgal)
- Less sensitive to horizontal vibrations
- Requires a constant temperature environment

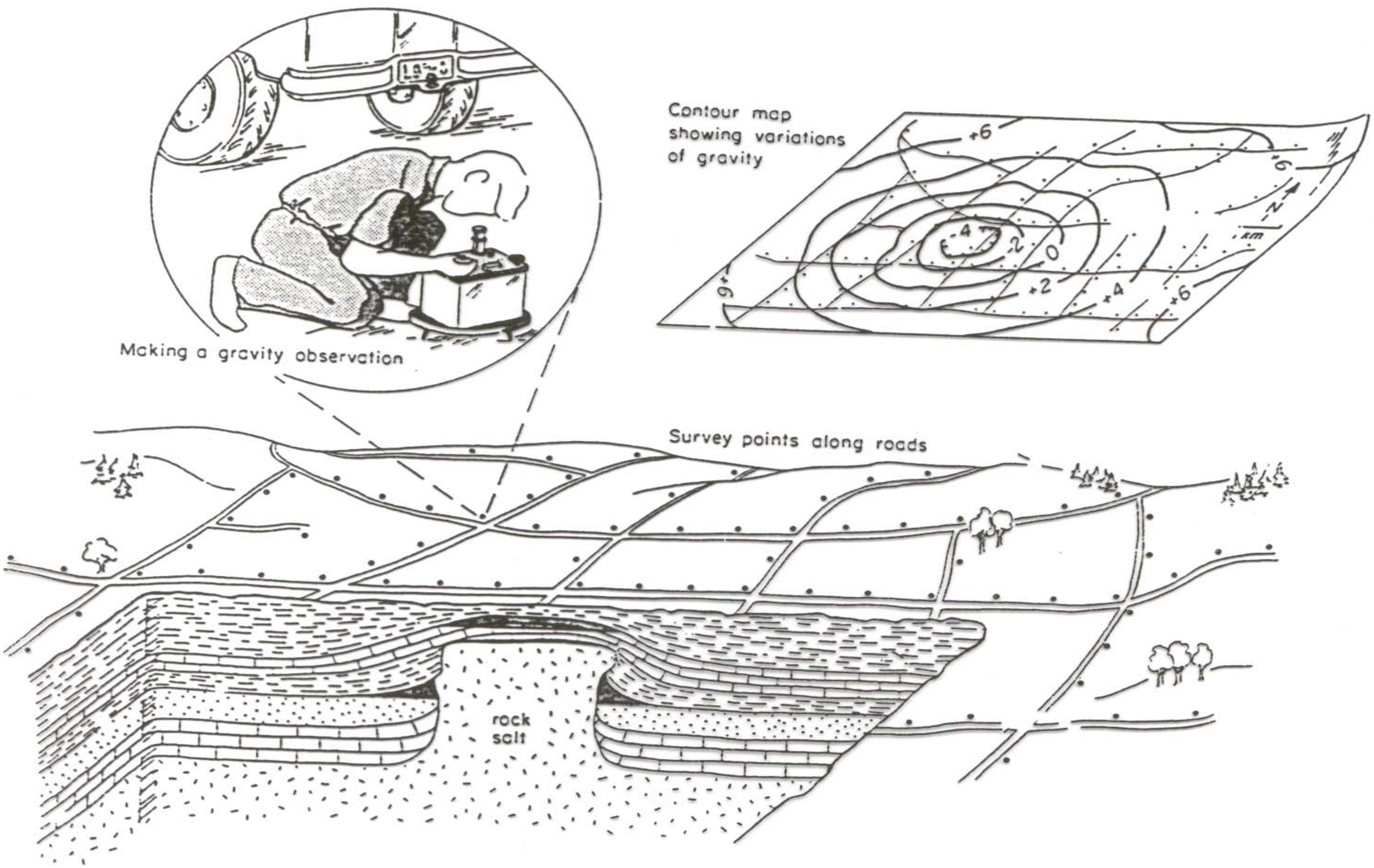
CG-5 Autograv

CG-5 electronic gravimeter:

CG-5 gravimeter uses a mass supported by a spring. The position of the mass is kept fixed using two capacitors. The *dV* used to keep the mass fixed is proportional to the gravity.

- Self levelling
- Rapid measurement rate (6 meas/sec)
- Filtering
- Data storage

Gravity surveying



Factors that influence gravity

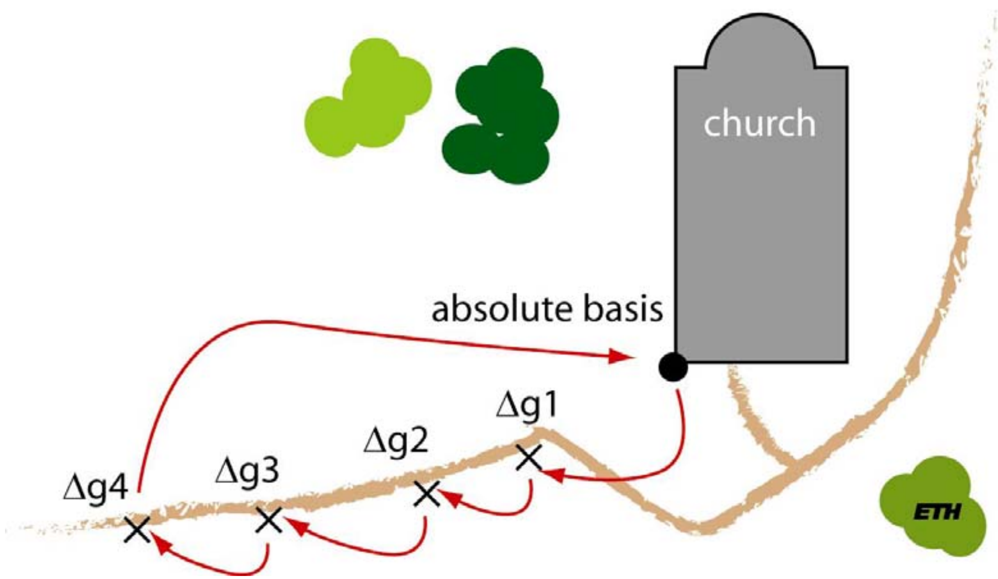
The magnitude of gravity depends on 5 factors:

- Latitude
- Elevation
- Topography of the surrounding terrains
- Earth tides
- Density variations in the subsurface:

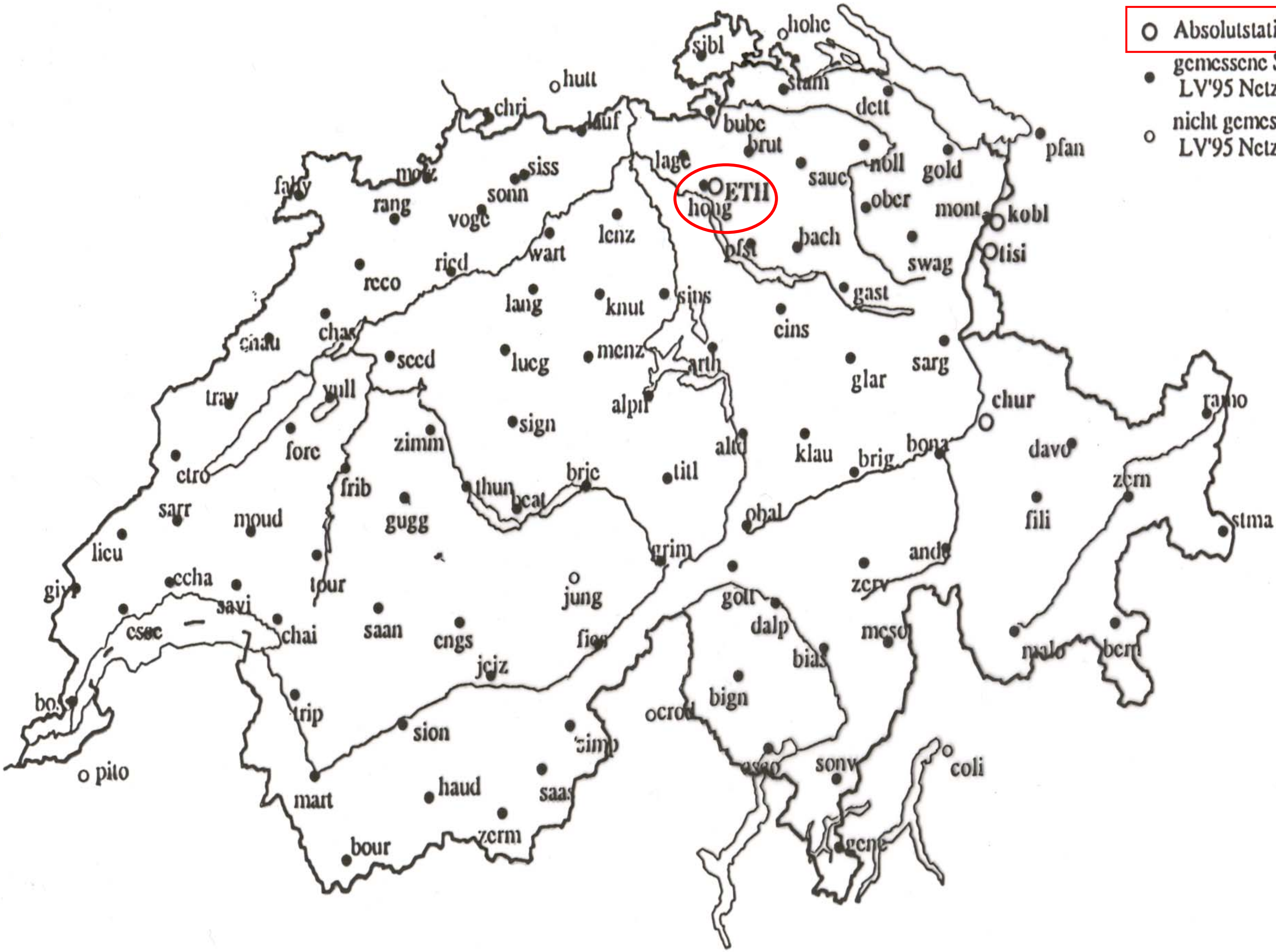
  this is the factor of interest in gravity exploration, but it is much smaller than latitude or elevation effects!

Gravity surveying

- Good location is required (about 10m)
- Uncertainties in elevations of gravity stations account for the greatest errors in reduced gravity values (precision required about 1 cm) (use dGPS)
- Frequently read gravity at a base station (looping) needed



Neues Schwerefundamentalnetz der Schweiz



Legende

- o Absolutstationen
- • gemessene Stationen LV'95 Netz
- ∘ nicht gemessene Stationen LV'95 Netz

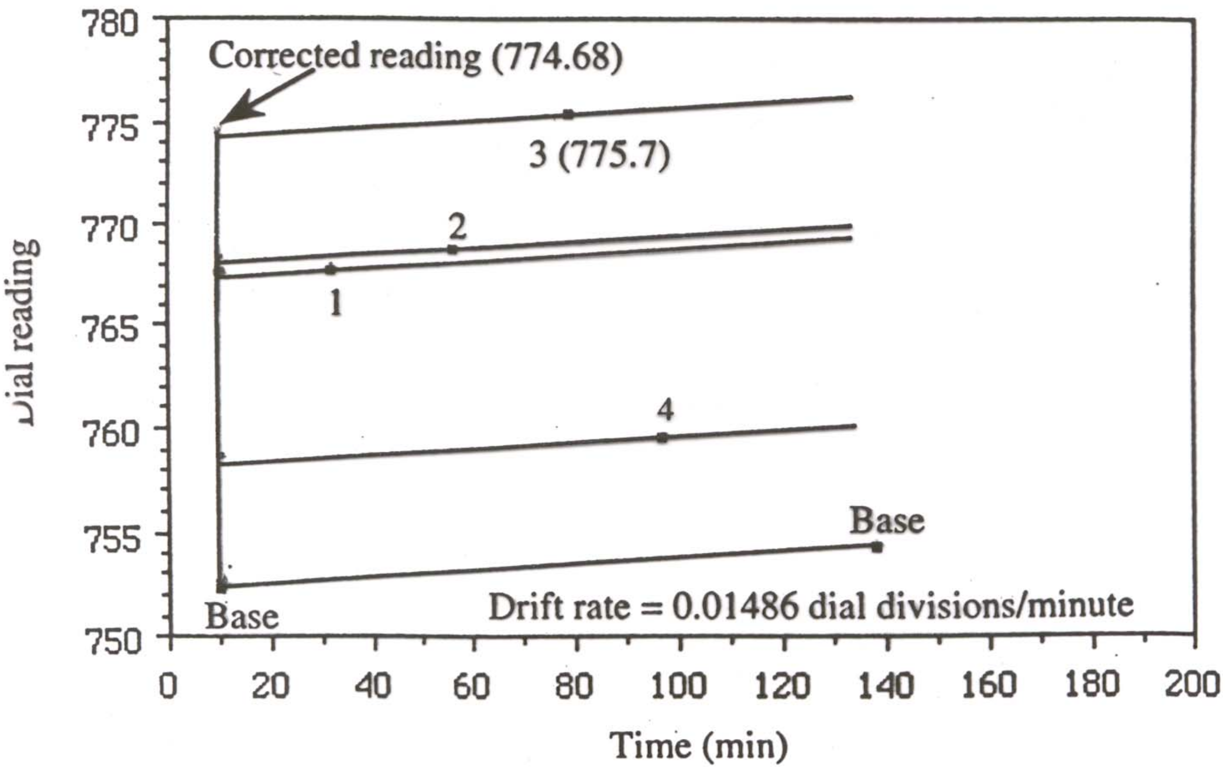
Observed data corrections

g_{obs} can be computed for the stations using Δg only after the following corrections:

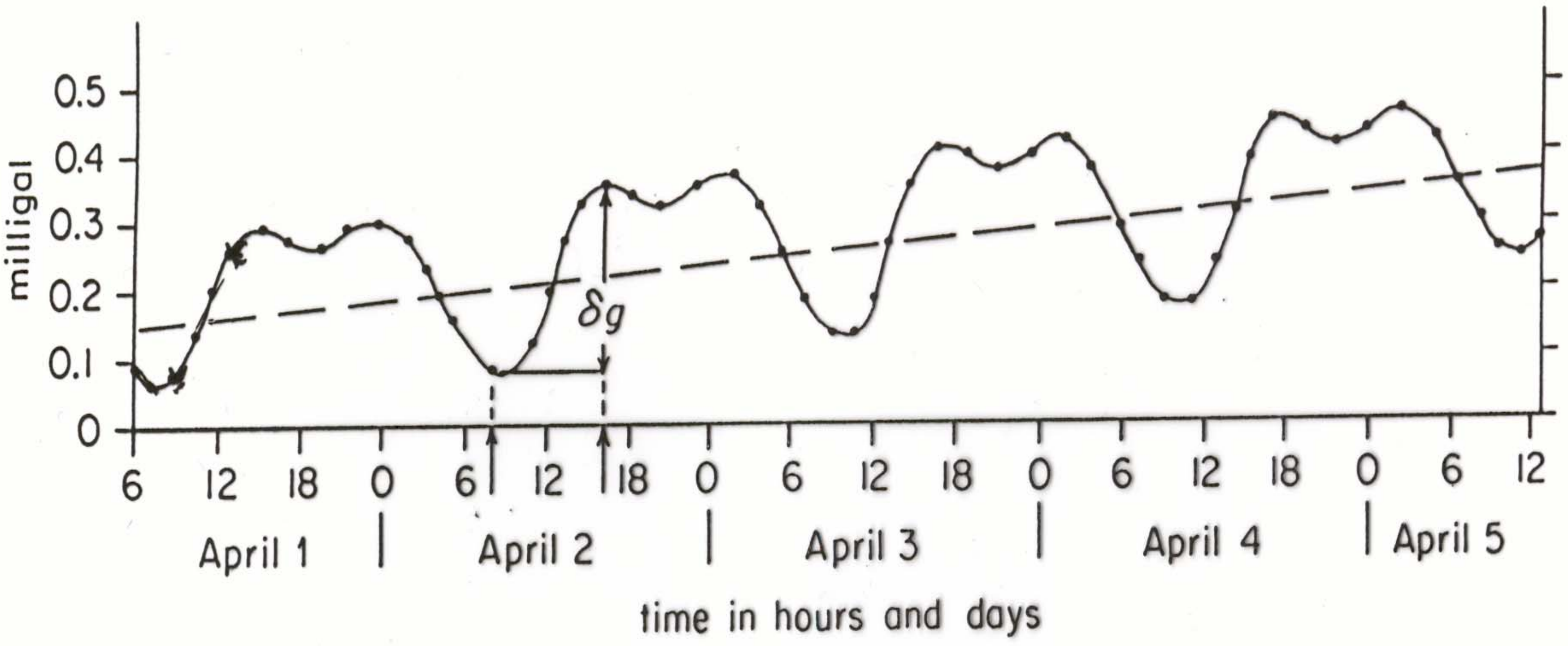
- Drift correction
- Tidal correction
- Distance ground/gravimeter („free air correction“ see below)

Drift correction on observed data

Gradual linear change in reading with time, due to imperfect elasticity of the spring (creep in the spring)



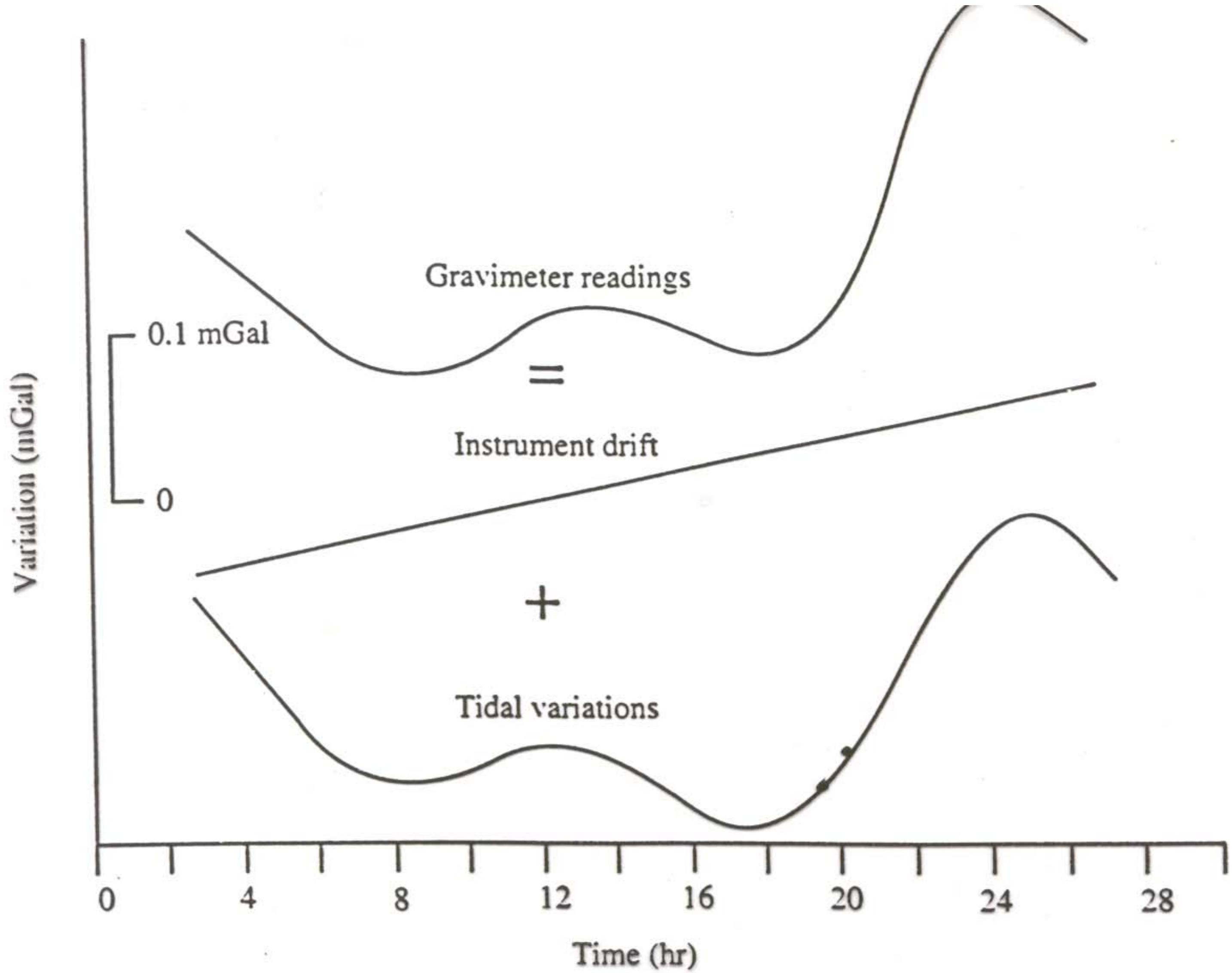
Tidal correction on observed data



Effect of the Moon: about 0.1 mgal

Effect of the Sun: about 0.05 mgal

After drift and tidal corrections, g_{obs} can be computed using Δg, the calibration factor of the gravimeter and the value of gravity at the base



Gravity reduction: Bouguer anomaly

$$BA = g_{obs} - g_{model}$$

$$g_{model} = g_{\phi} - FAC + BC - TC$$

- g_{model} model for an on-land gravity survey
- g_{ϕ} gravity at latitude ϕ (latitude correction)
- FAC free air correction
- BC Bouguer correction
- TC terrain correction

Latitude correction

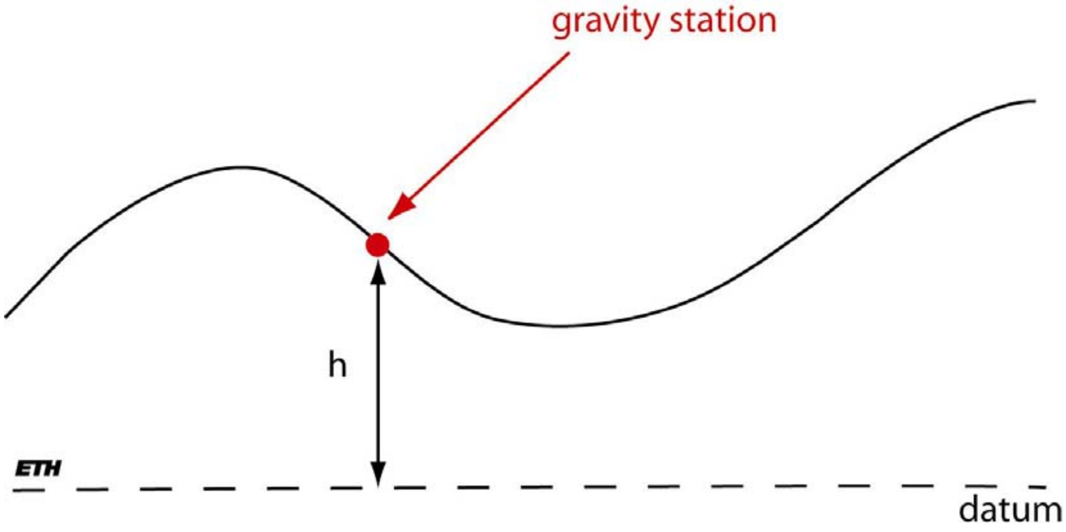
$$g_{\phi} = g_{equator}\left(1 + \beta_1 \sin^2\phi + \beta_2 \sin^4\phi\right)$$

- β_1 and β_2 are constants dependent on the shape and speed of rotation of the Earth
- The values of β_1, β_2 and $g_{equator}$ are definded in the Gravity Formula 1967 (reference spheroid)

Free air correction

The *FAC* accounts for variation in the distance of the observation point from the centre of the Earth.

This equation must also be used to account for the distance ground/gravimeter.



Free air correction

$$g = \frac{GM}{R^2}$$

$$\frac{dg}{dR} = -2\frac{GM}{R^3} = -2\frac{g_N}{R}$$

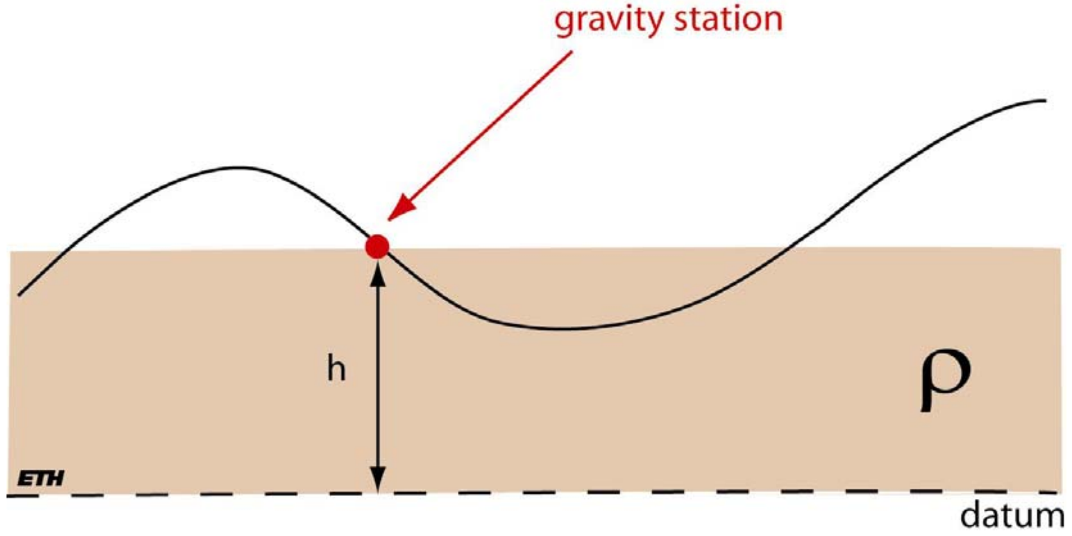
$$\Delta g_{Höhe} \approx 2\frac{g_N dR}{R} \approx 0.3\,\mathrm{mgal} \cdot dR$$

$$FAC = 0.3086\,h \quad (h \text{ in meters})$$

Bouguer correction

- The *BC* accounts for the gravitational effect of the rocks present between the observation point and the datum
- Typical reduction density for the crust is ρ =2.67 g/cm^3

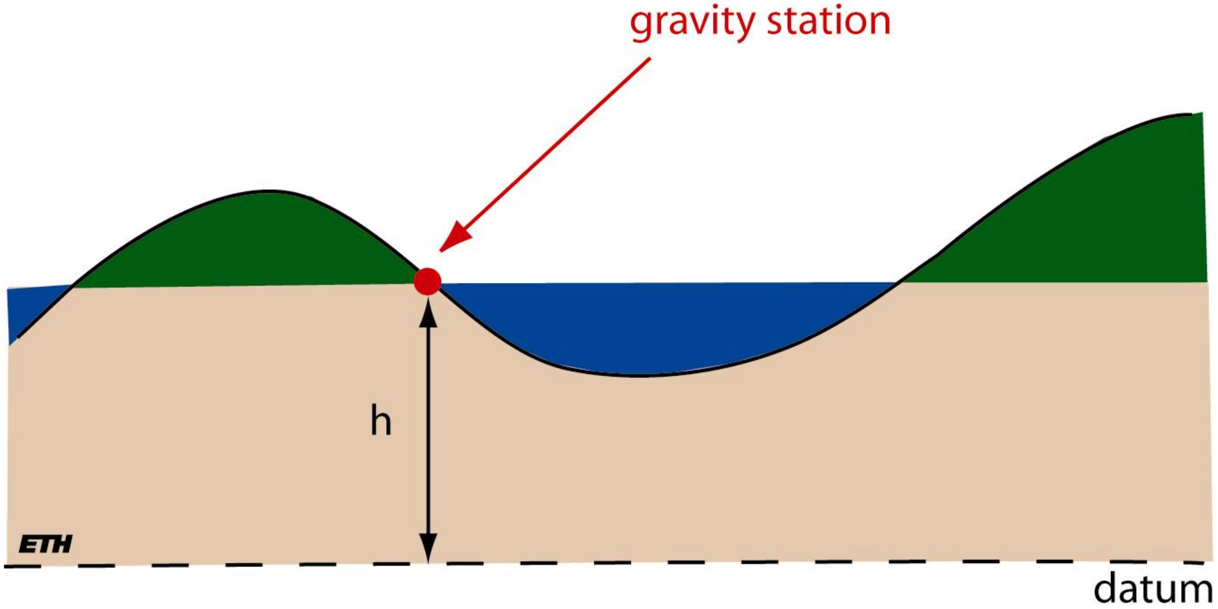
$$BC = 2\pi\, G\, \rho\, h$$

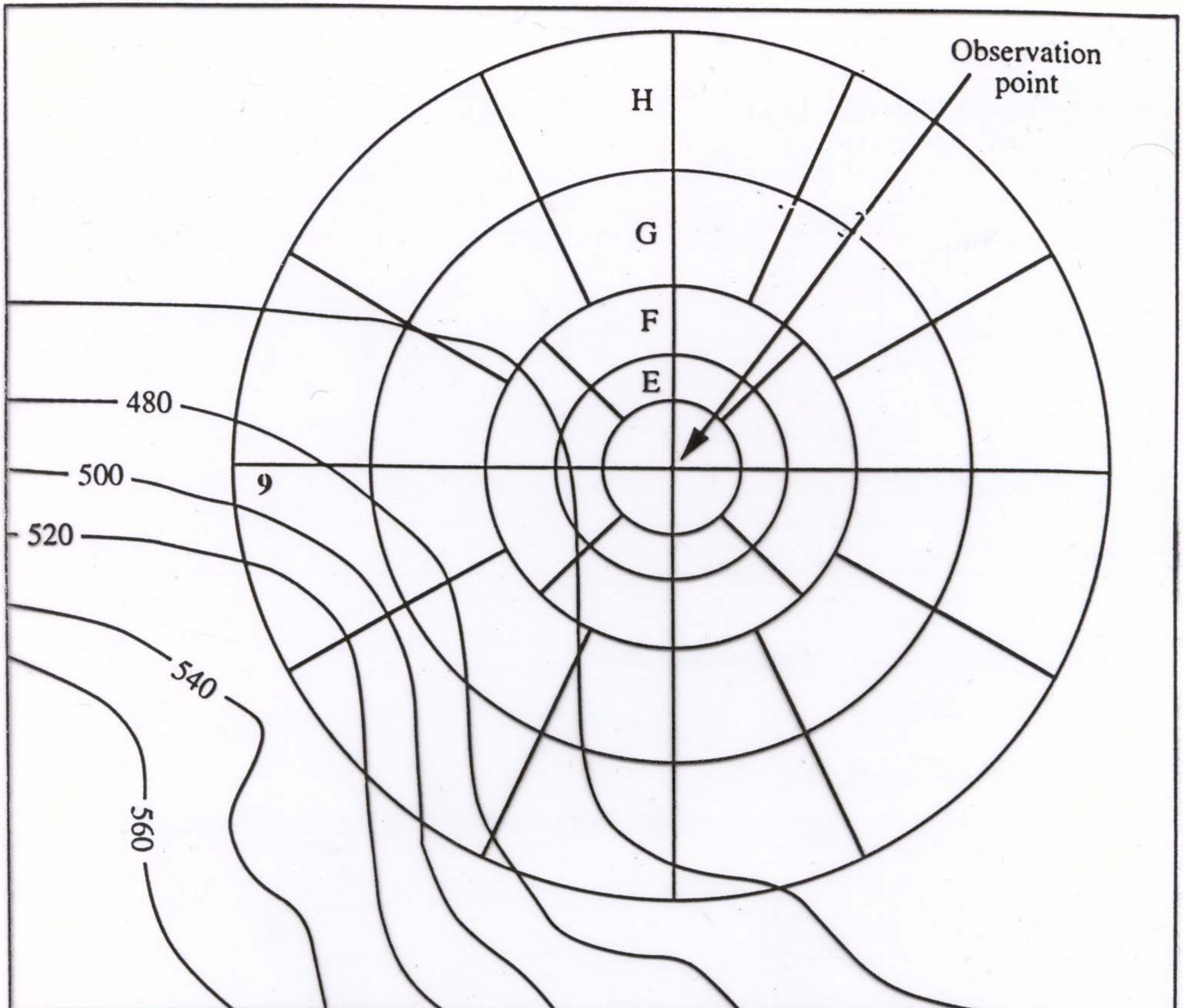


Terrain correction

The *TC* accounts for the effect of topography.

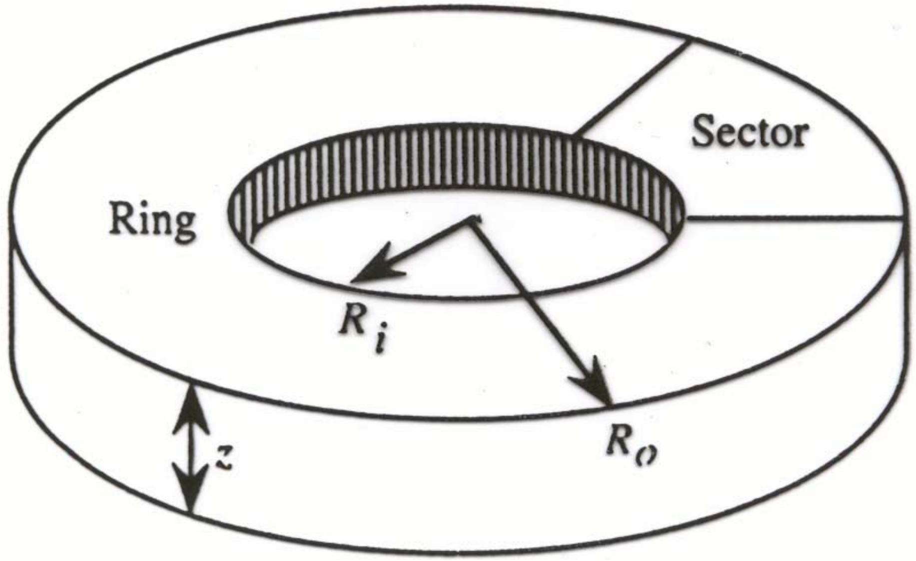
The terrains in green and blue are taken into account in the *TC* correction in the same manner: why?



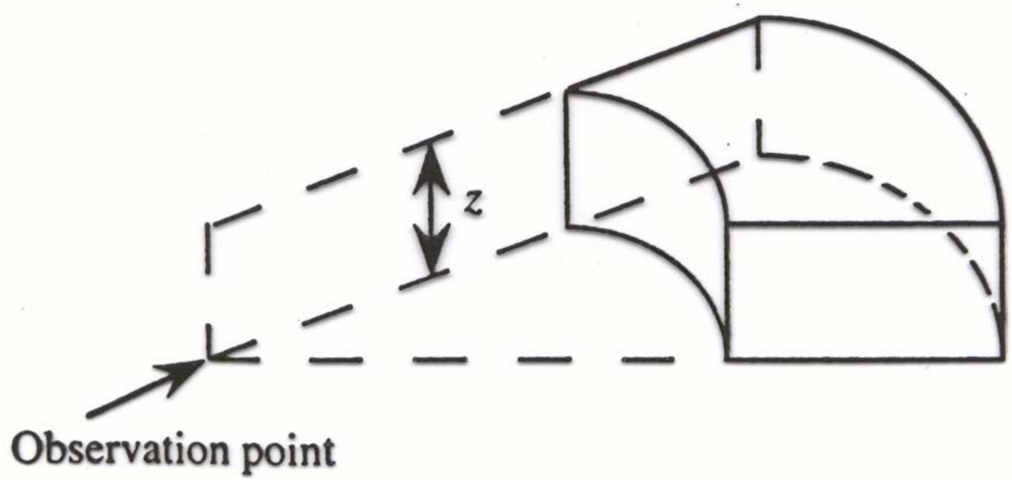


Observation point
H
G
F
E
9
480
500
520
540
560

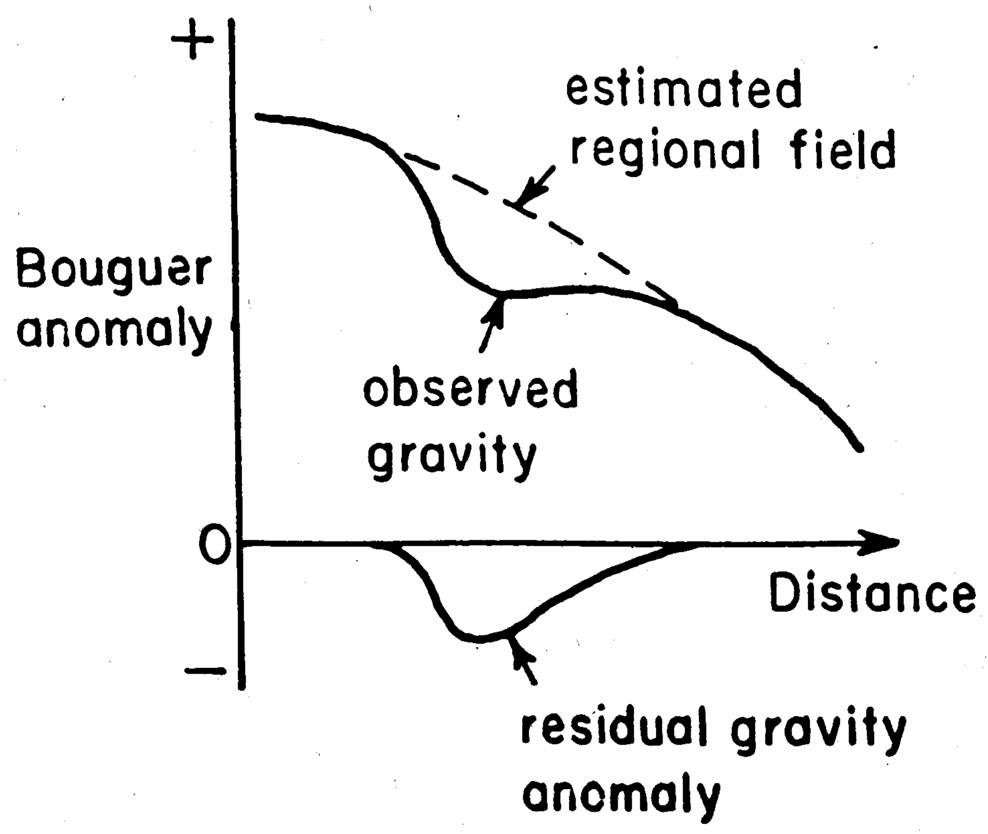
(a)



(b)

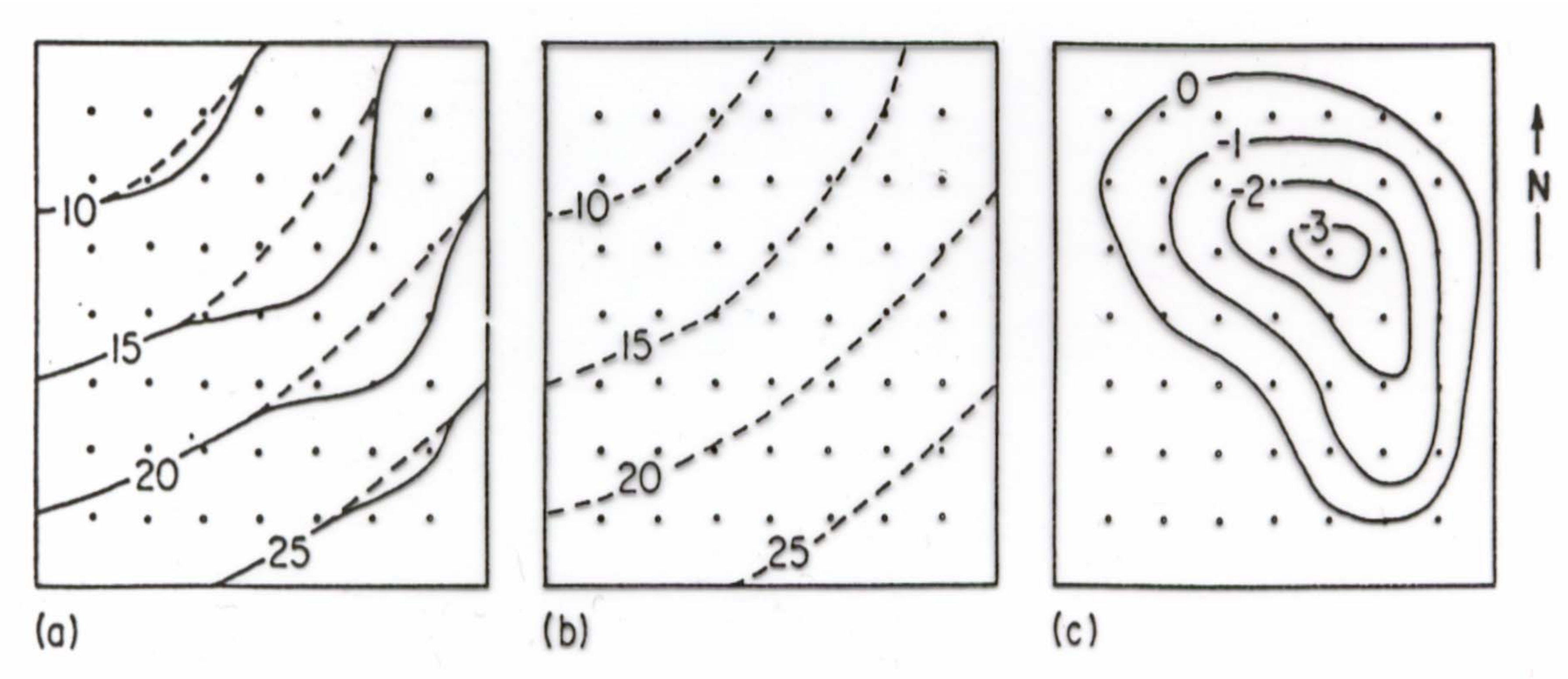


Residual gravity anomaly

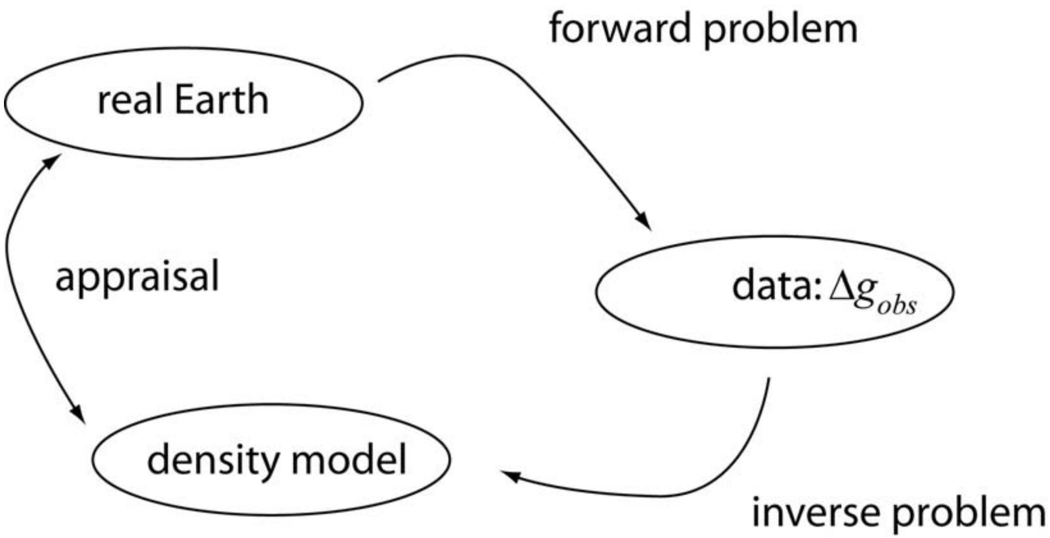


The regional field can be estimated by hand or using more elaborated methods (e.g. upward continuation methods)

Bouguer anomaly



Interpretation: the inverse problem

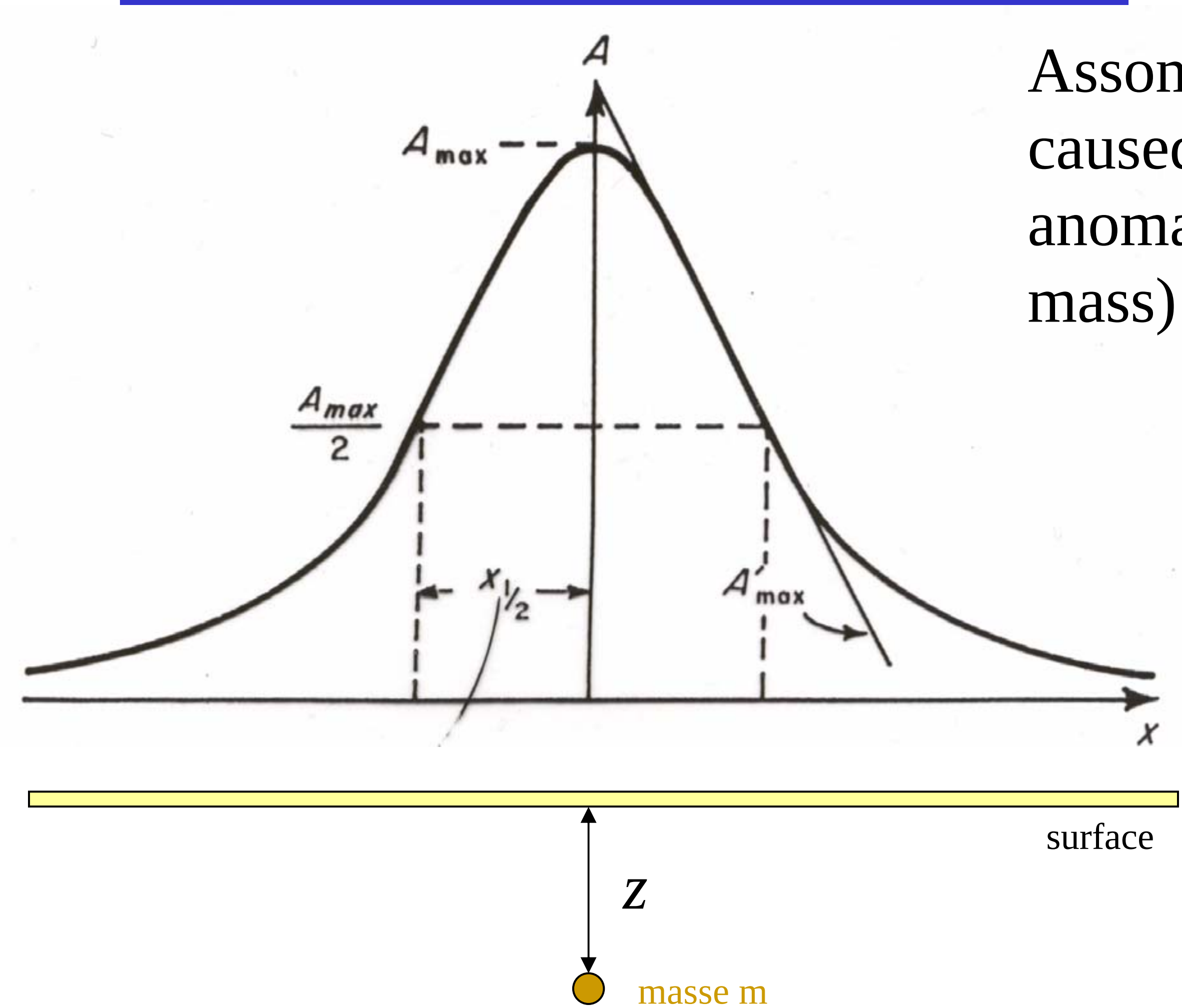


Two ways of solving the inverse problem:

- „Direct“ interpretation
- „Indirect“ interpretation and automatic inversion

Warning: „direct“ interpretation has nothing to do with „direct“ (forward) problem!

Direct interpretation



Assomption: a 3D anomaly is caused by a point mass (a 2D anomaly is caused by a line mass) at depth=z

$x_{1/2}$ gives z

Direct interpretation

Geometry	*Formula*	*Depth*
Ball	$\Delta g = \frac{4\pi G R^3 \Delta\rho}{3z^3} \frac{1}{\left[1 + \left(x^2 / z^2\right)\right]^{3/2}}$	$z = 1.305 x_{1/2}$
Horizontal cylinder	$\Delta g = \frac{2\pi G R^2 \Delta\rho}{z} \frac{1}{\left[1 + \left(x^2 / z^2\right)\right]}$	$z = 1.0 x_{1/2}$
Vertical cylinder	$\Delta g = \frac{\pi G R^2 \Delta\rho}{\left(x^2 + z^2\right)^{1/2}}$	$z = 0.58 x_{1/2}$

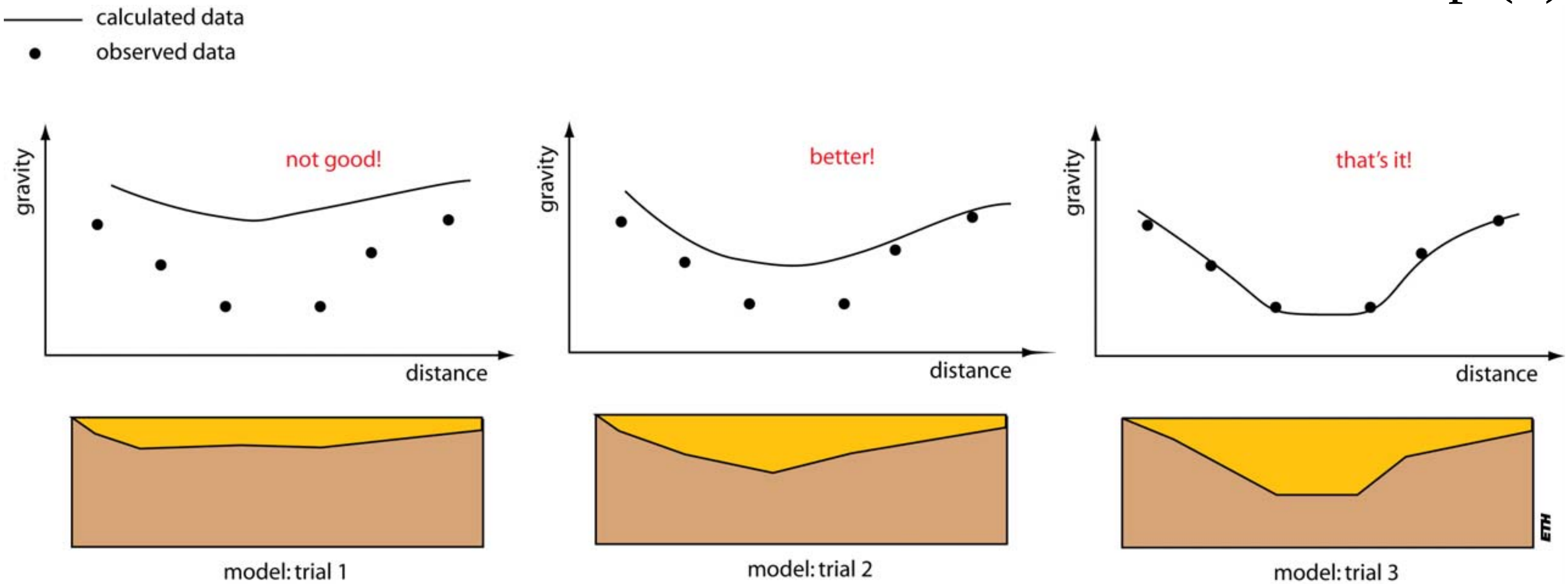
Indirect interpretation

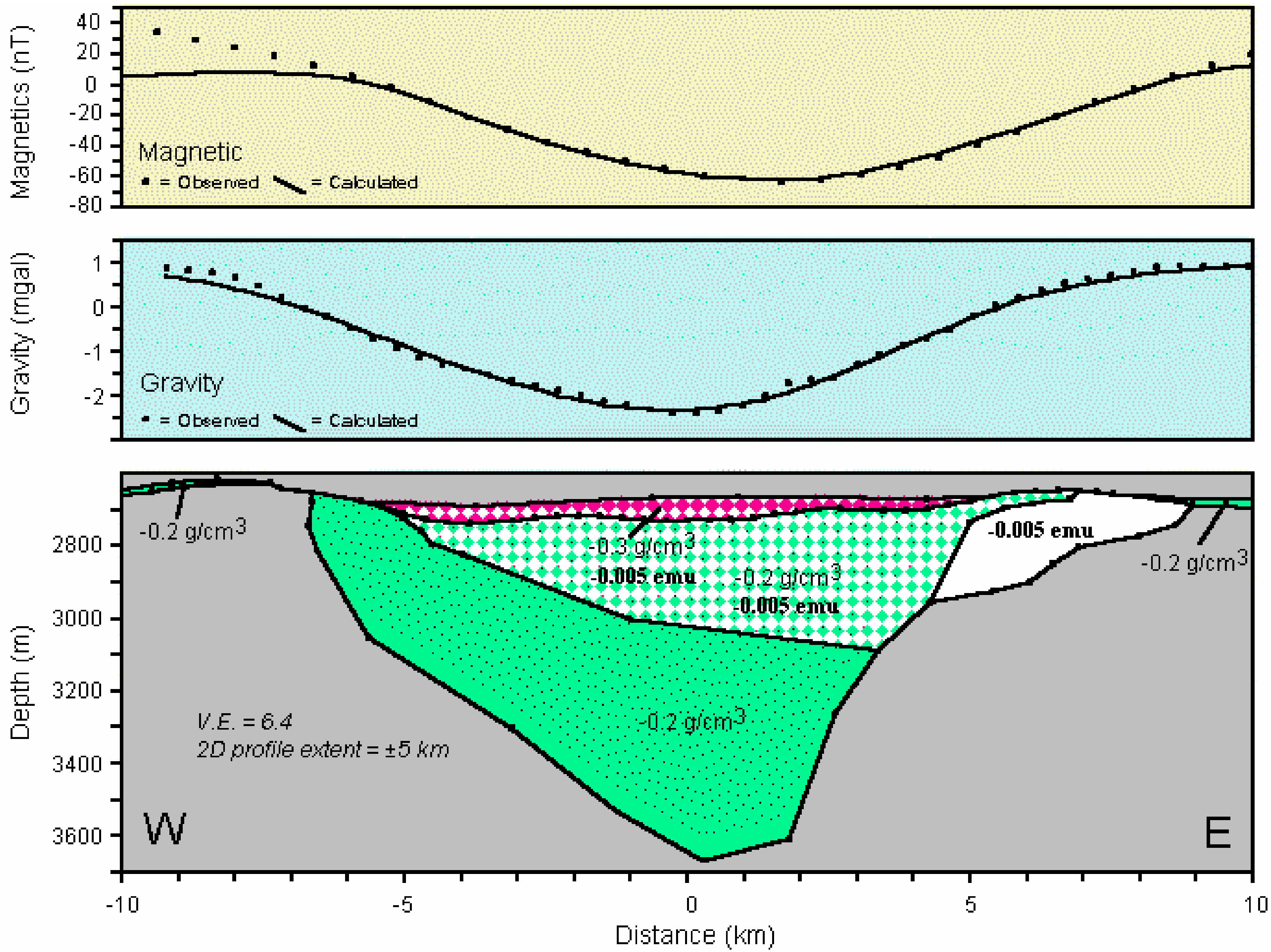
(1) Construction of a reasonable model

(2) Computation of its gravity anomaly

(3) Comparison of computed with observed anomaly

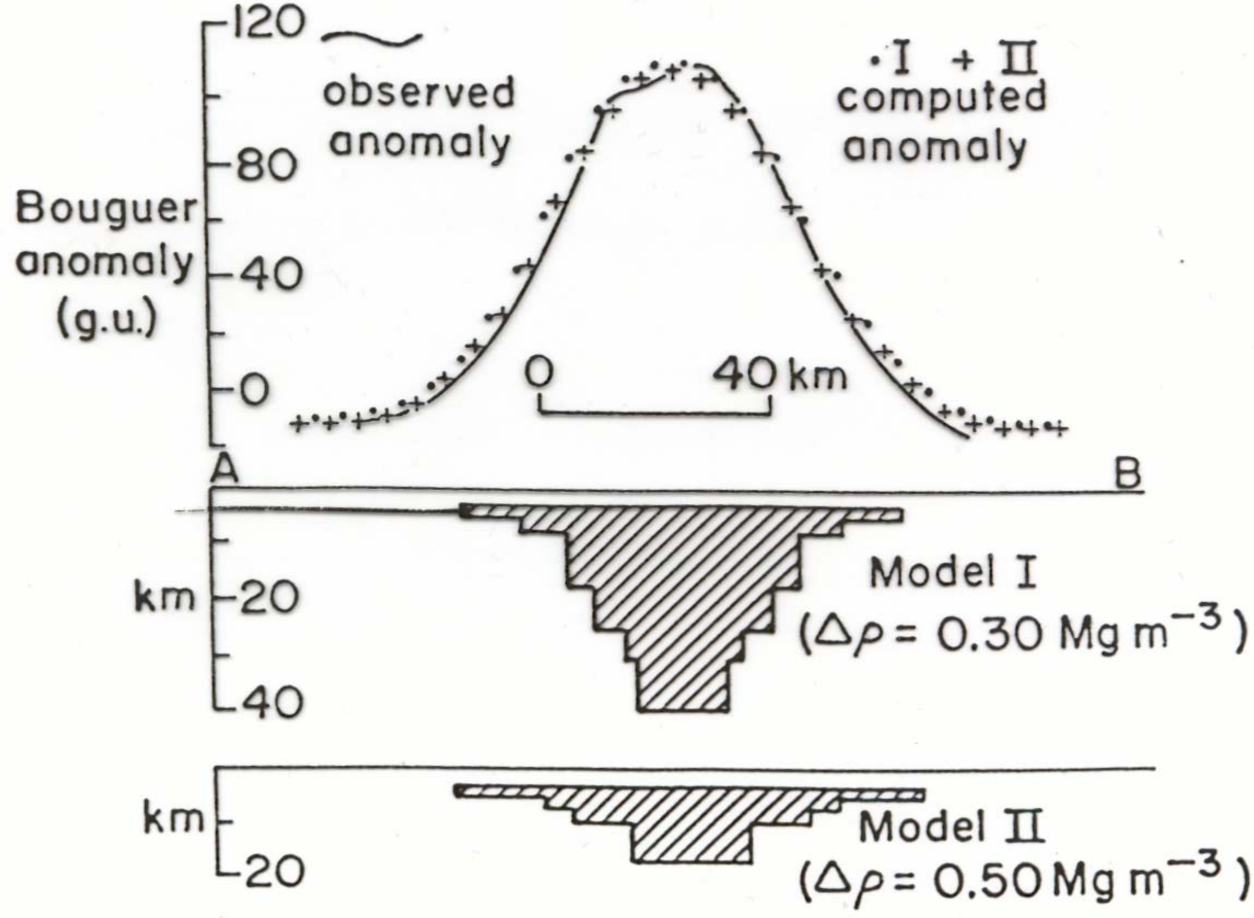
(4) Alteration of the model to improve correspondence of observed and calculated anomalies and return to step (2)





Magnetics (nT)
40
20
0
-20
-40
-60
-80
Magnetic
■ = Observed = Calculated
Gravity (mgal)
1
0
-1
-2
Gravity
■ = Observed = Calculated
Depth (m)
2800
3000
3200
3400
3600
-0.2 g/cm3
-0.3 g/cm3
-0.005 emu
-0.2 g/cm3
-0.005 emu
-0.005 emu
-0.2 g/cm3
-0.2 g/cm3
V.E. = 6.4
2D profile extent = ±5 km
W
E
-10
-5
0
5
10
Distance (km)

Non-unicity of the solution



Automatic inversion

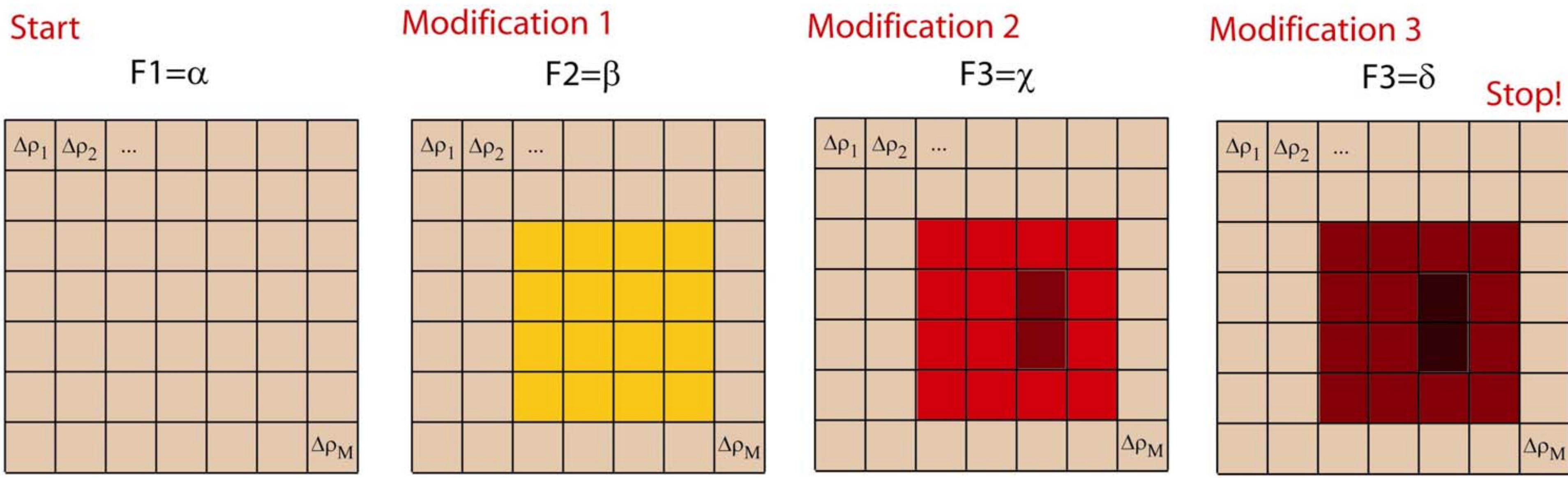
Automatic computer inversion with a priori information for more complex models (3D) using optimization algorithms. Minimize a cost (error) function *F*

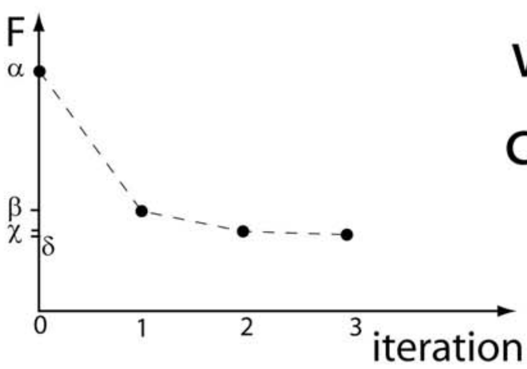
$$F = \sum_{i=1}^{n} \left(\Delta g_{obs_i} - \Delta g_{calc_i} \right)$$

with *n* the number of data

Automatic inversion is used when the model is complex (3D)

Automatic inversion

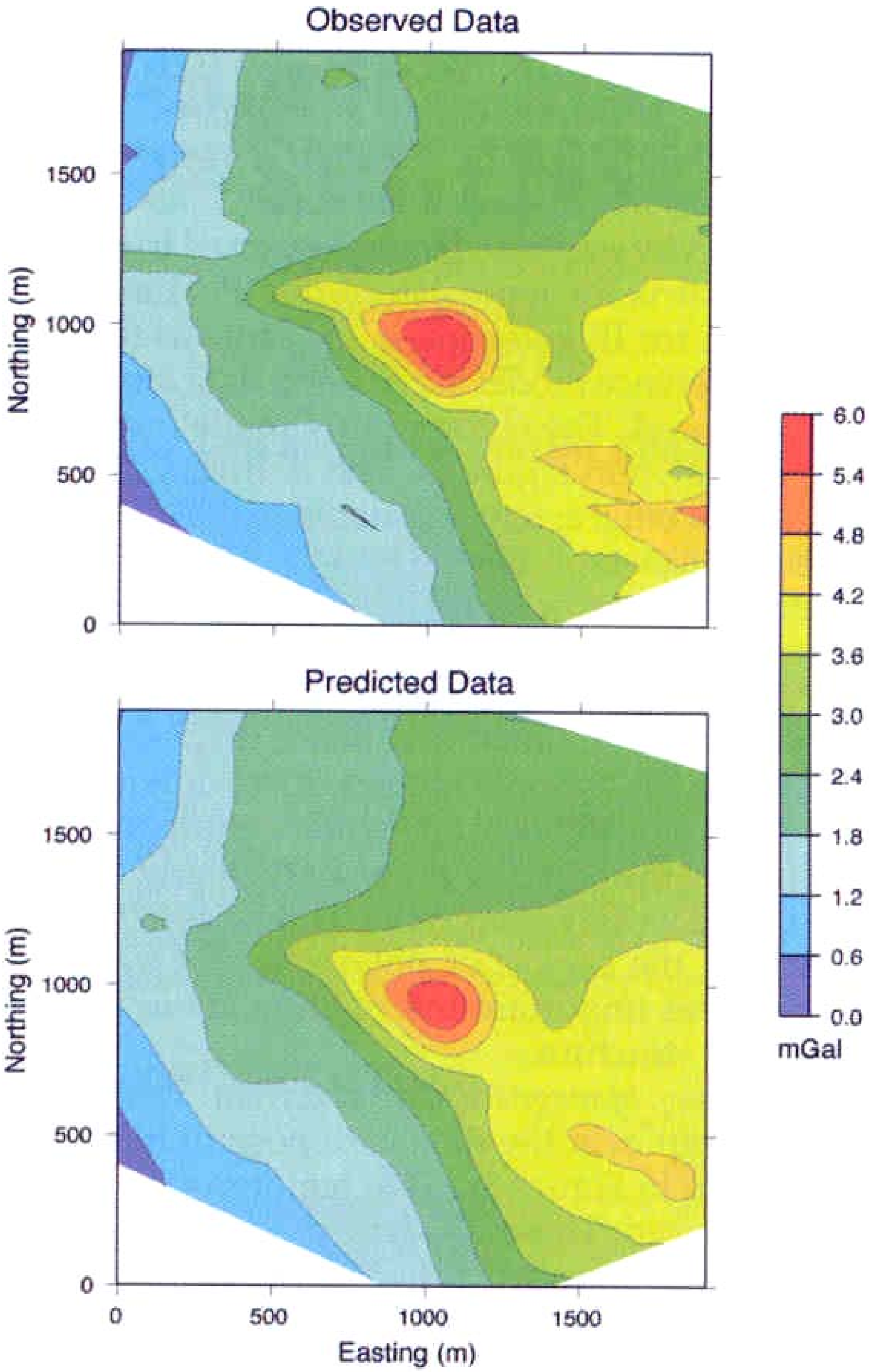


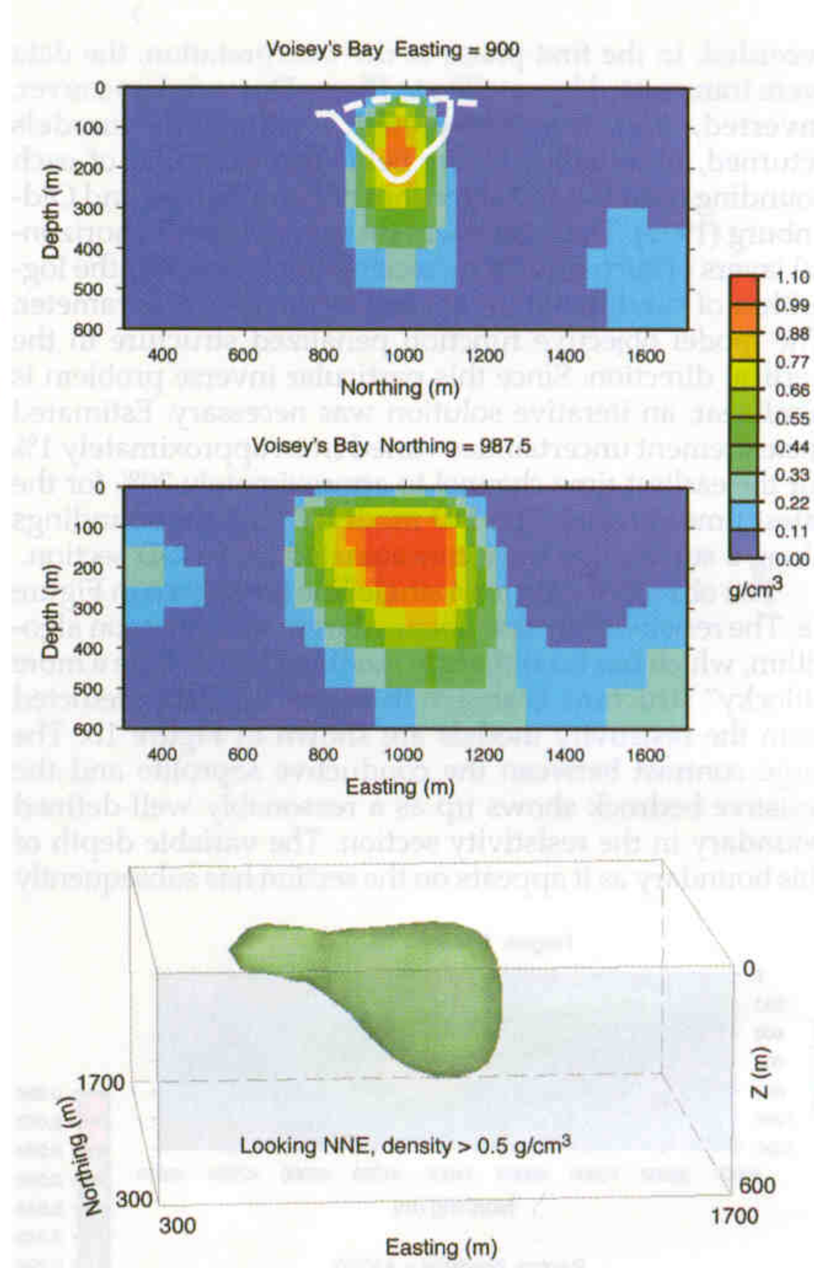


with $\alpha>\beta>\chi>\delta$

convergence and stop if $\chi \cong \delta$

Mining geophysics





5. Microgravity: a case history

A SUBWAY PROJECT IN LAUSANNE, SWITZERLAND, AS AN URBAN MICROGRAVIMETRY TEST SITE

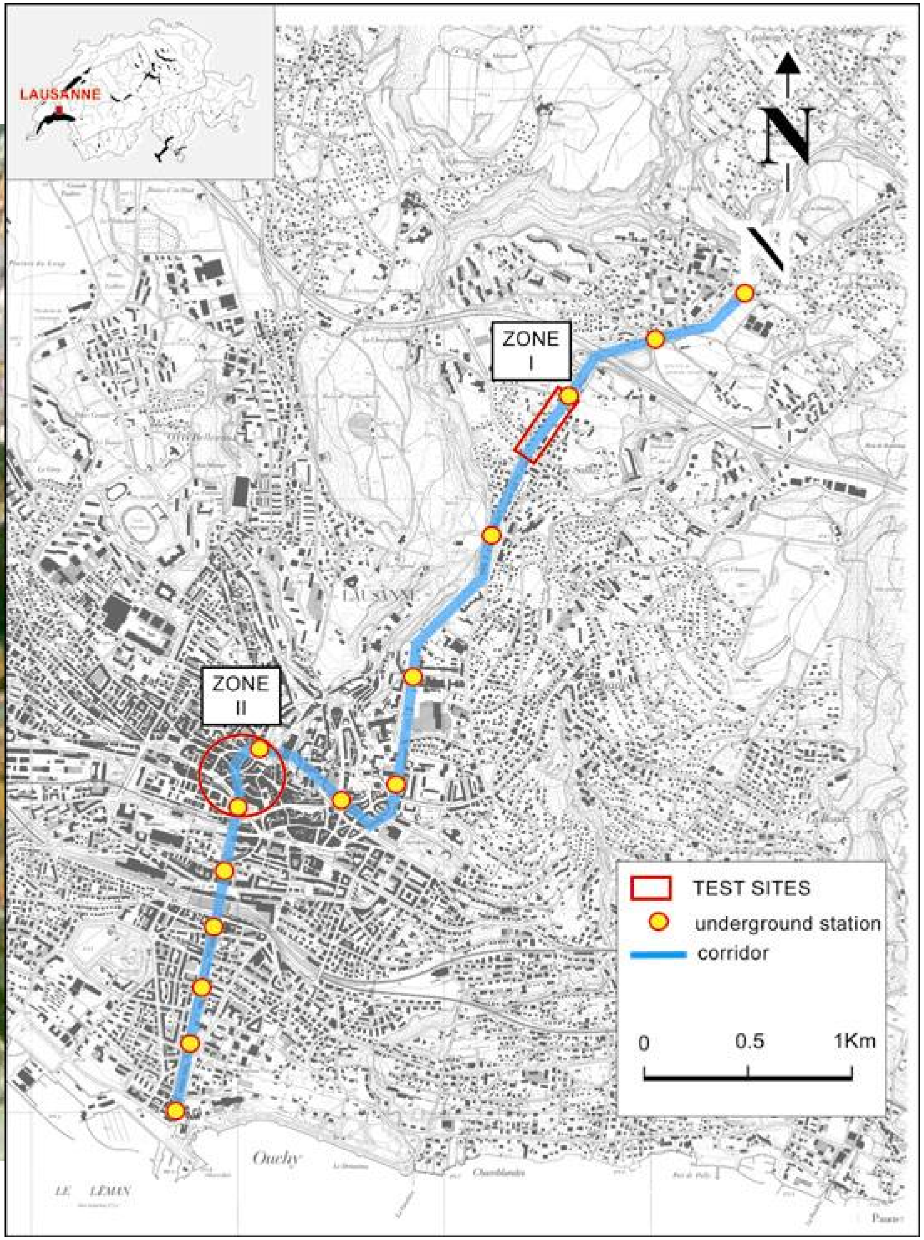
P. Radogna, R. Olivier, P. Logean and P. Chasseriau
Institute of Geophysics, University of Lausanne

- length:6 km
- difference in altitude: 323 m
- geology: alpine molassic bedrock (tertiary sandstone) and an overlaying quaternary glacial fill
- depth of bedrock: varying from 1.5 m to 25 m
- The choice of the corridor had to consider the depth of the bedrock

Source: P. Radogna et al.

Source: www.rodio.ch

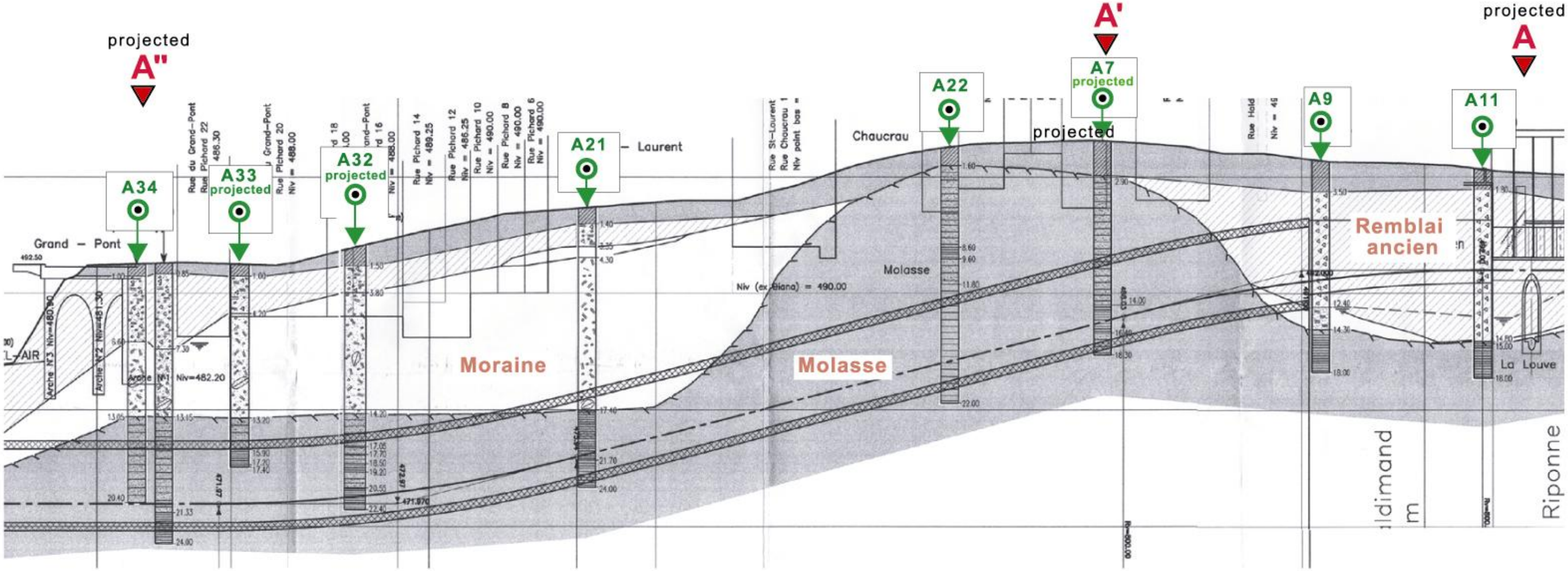
Zone II

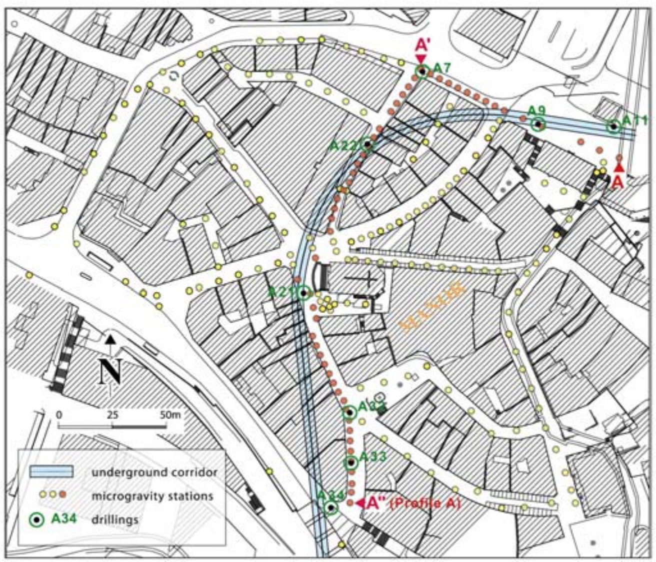


Scintrex CG5

200 gravity stations

Source: P. Radogna et al.



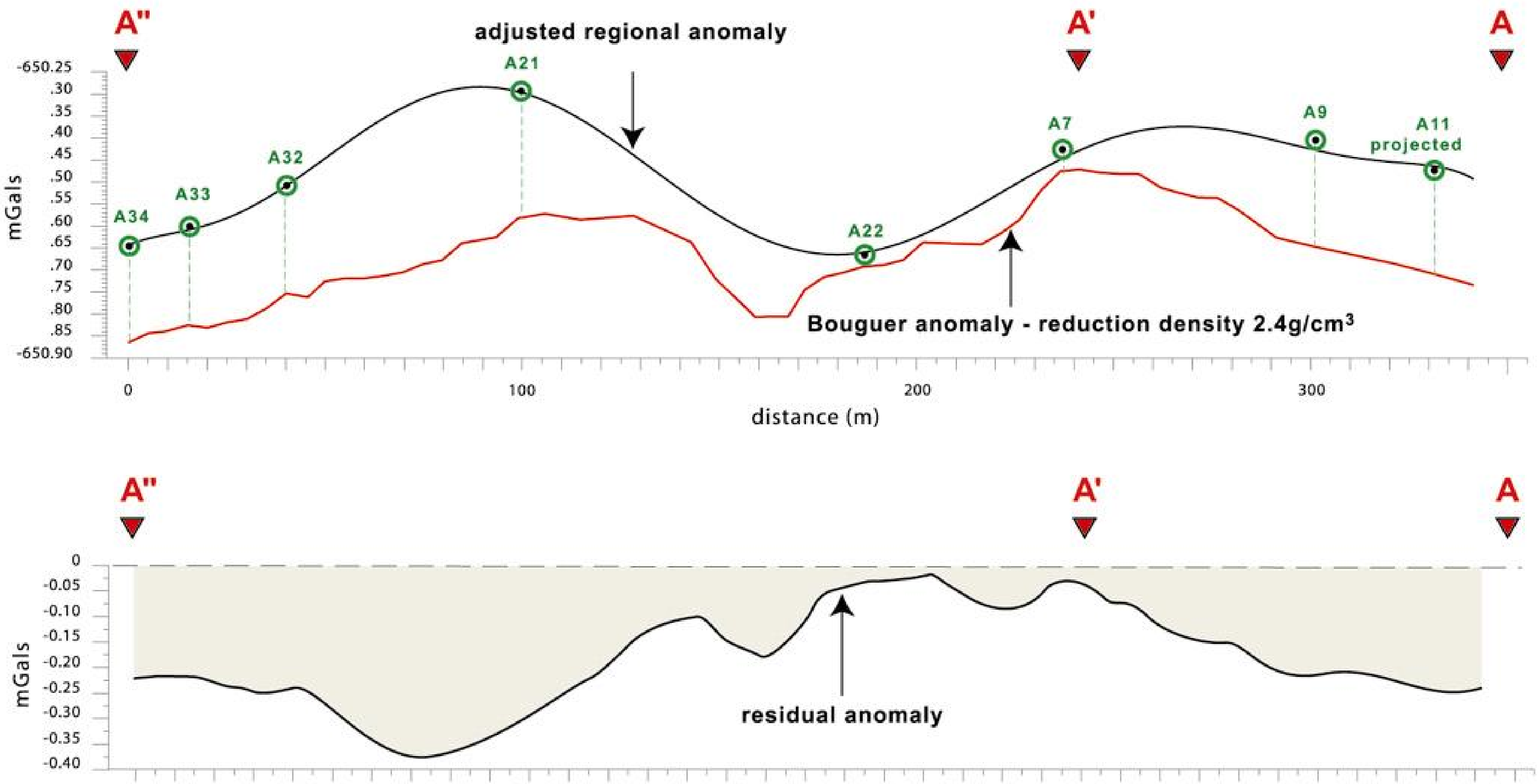


Geological section, approximately A´´-A´-A

Source: P. Radogna et al.

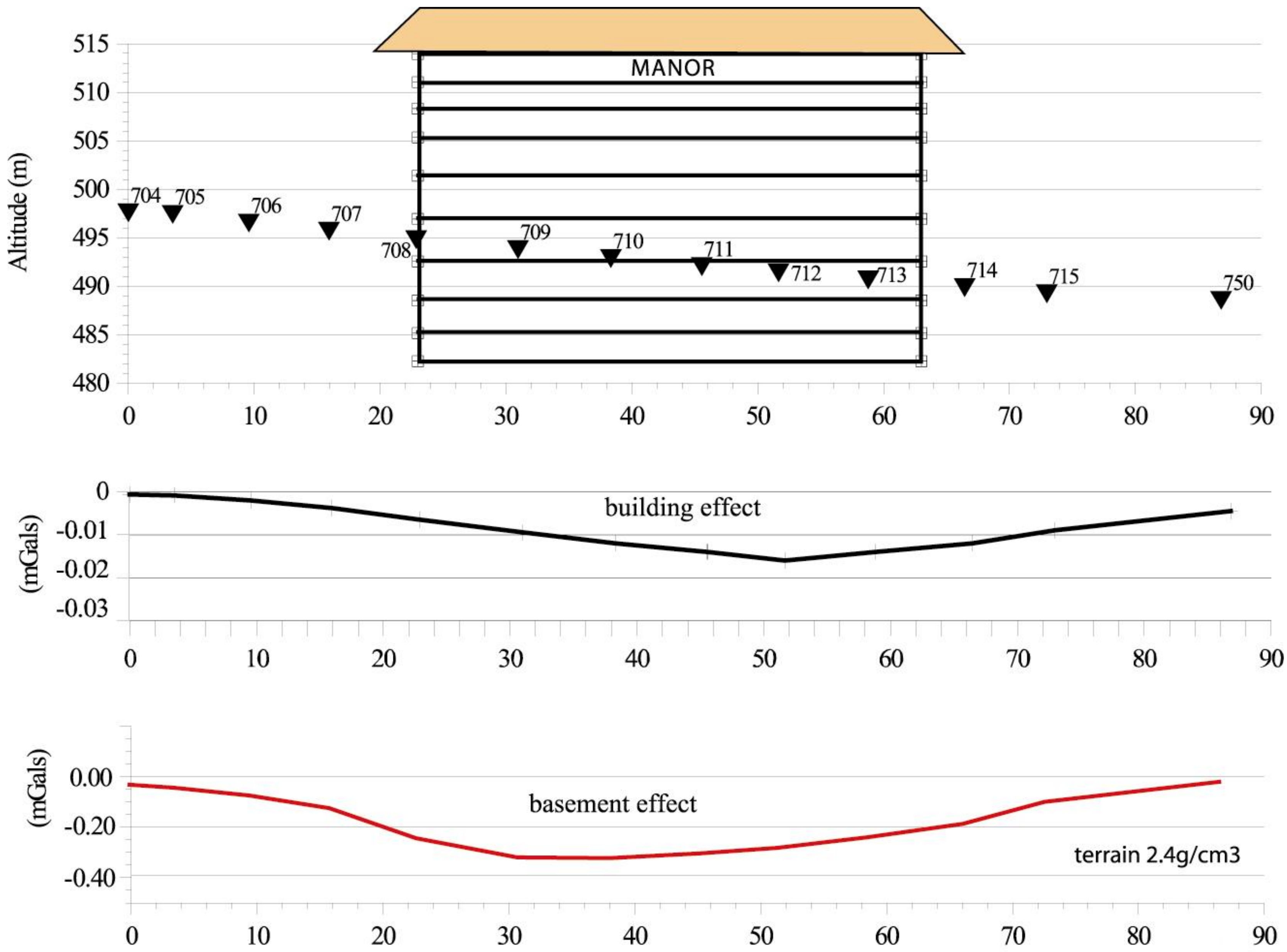
Source: P. Radogna et al.

Profile A´´-A´-A



Source: P. Radogna et al.

Building and basement gravity effect

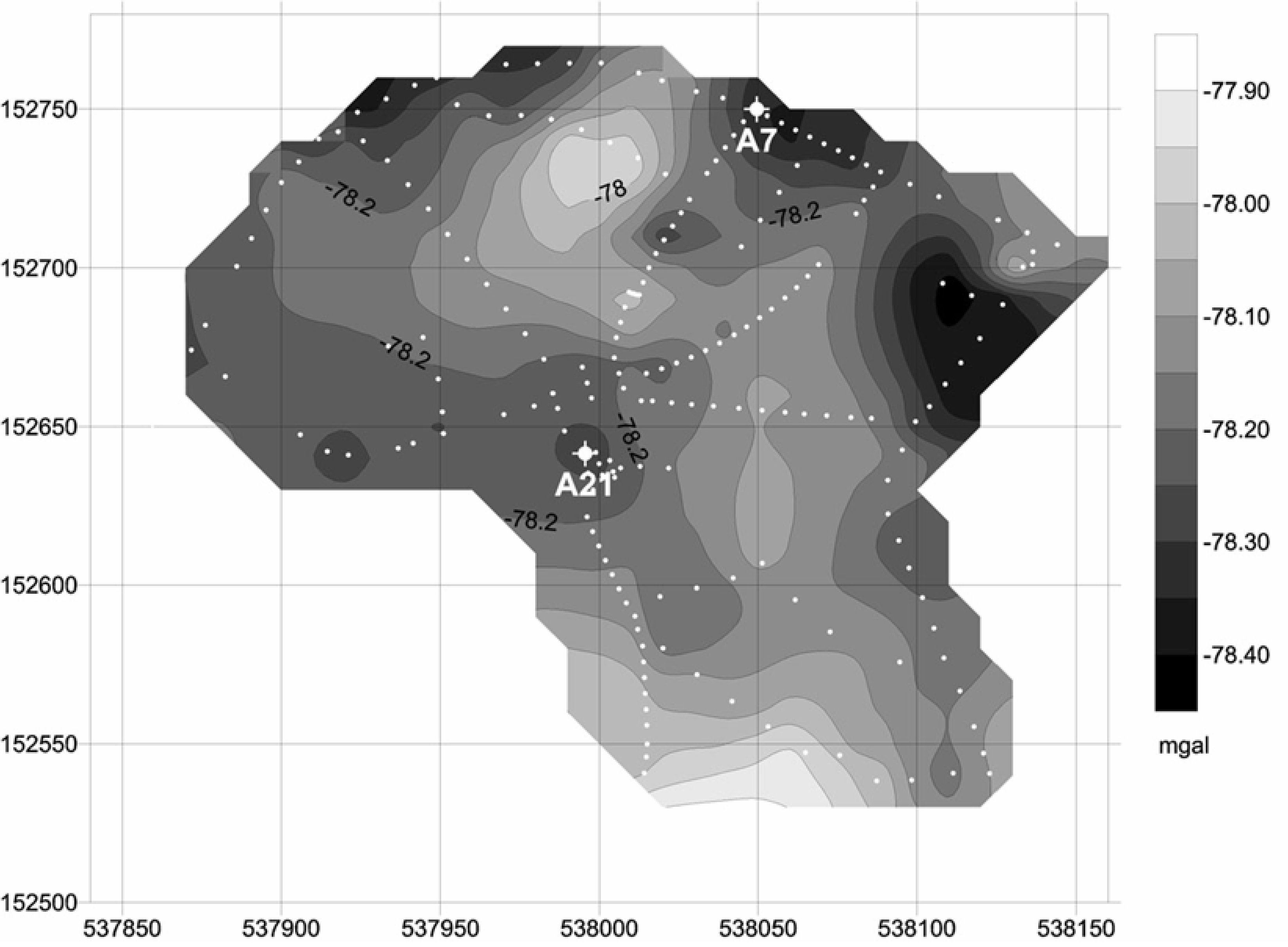


DEM for topographical corrections

BASEMENT FROM:
- Cadastral plan
- Building typology
- GIS

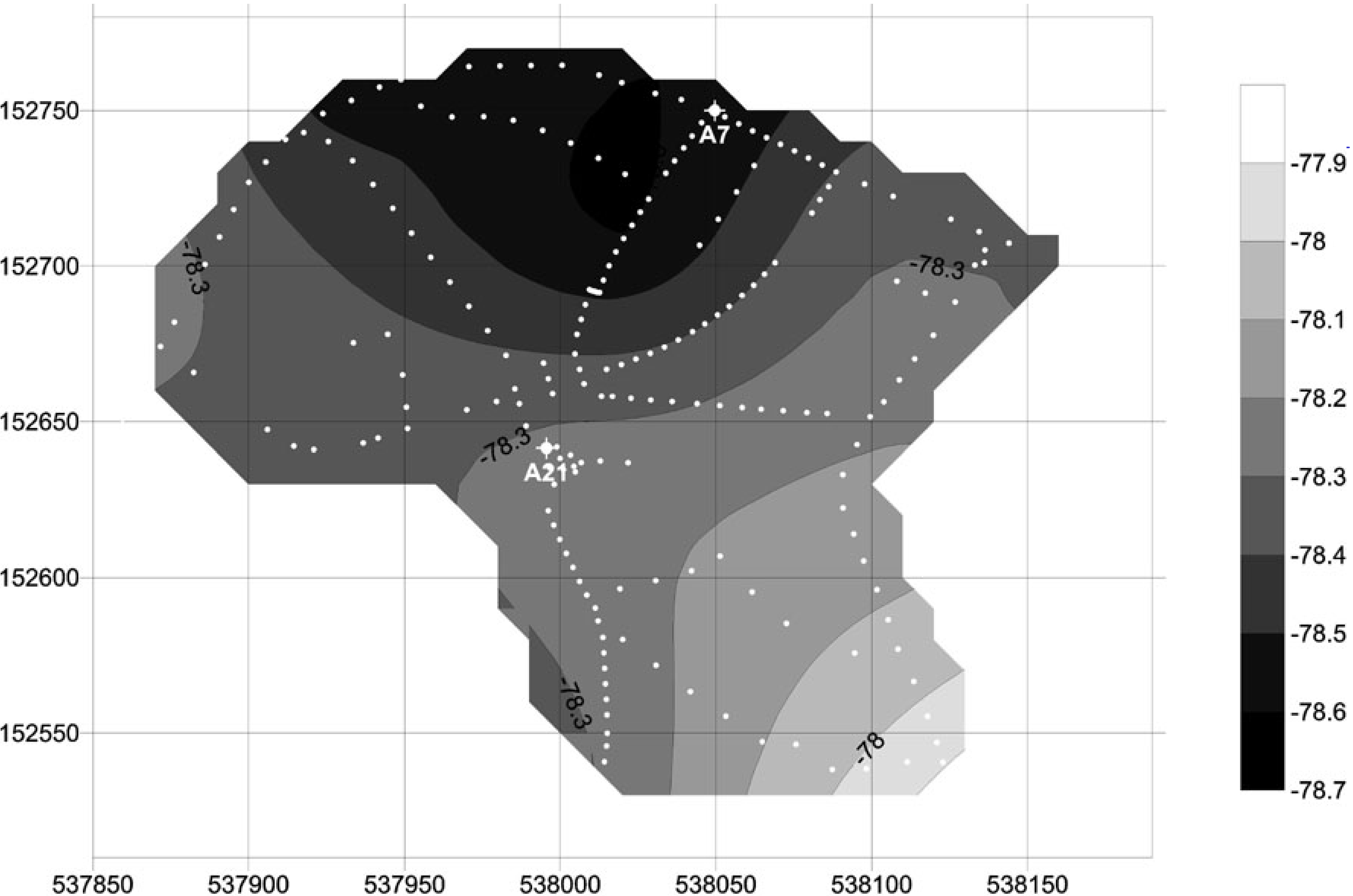
Source: P. Radogna et al.

Bouguer Anomaly



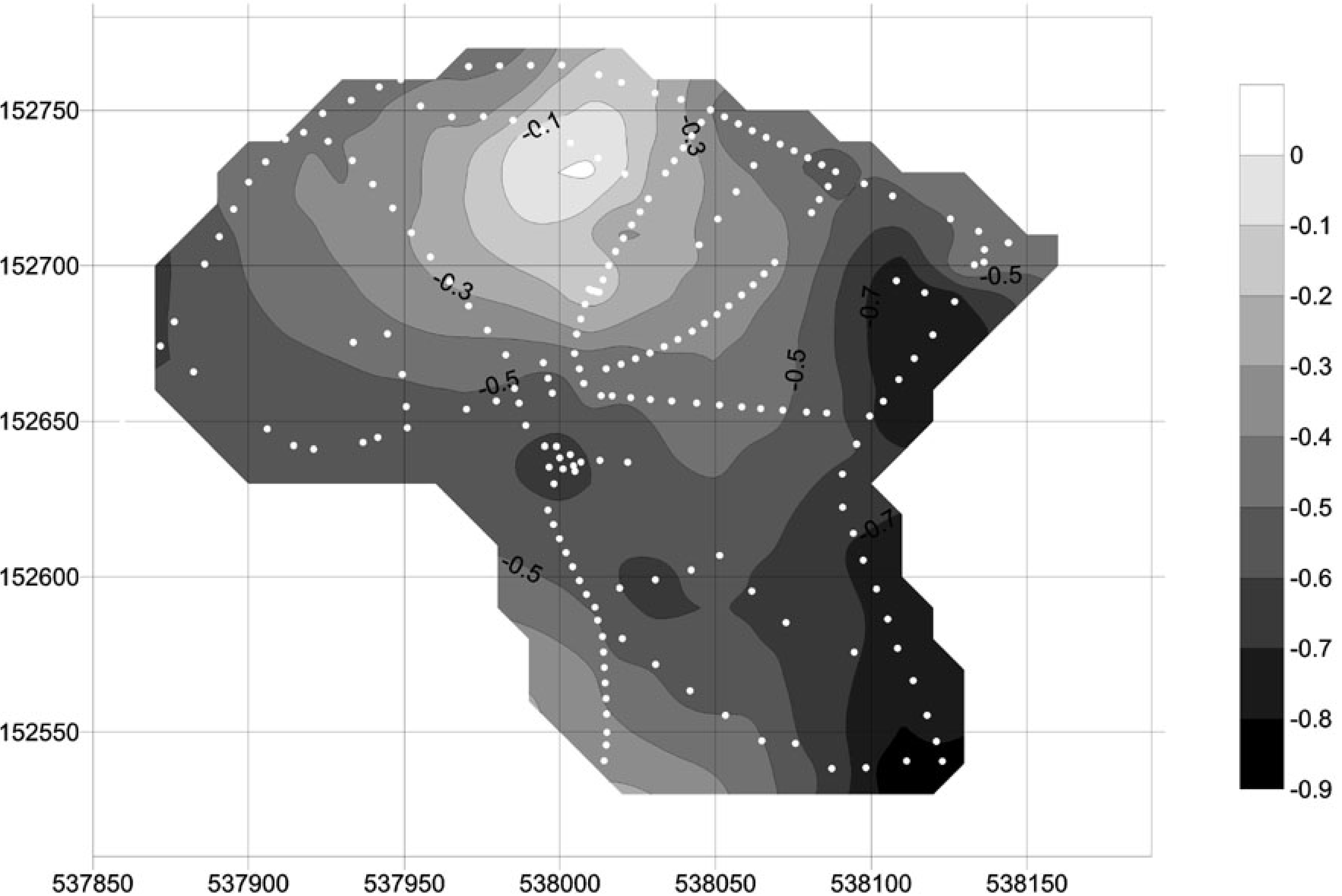
Source: P. Radogna et al.

Regional Anomaly



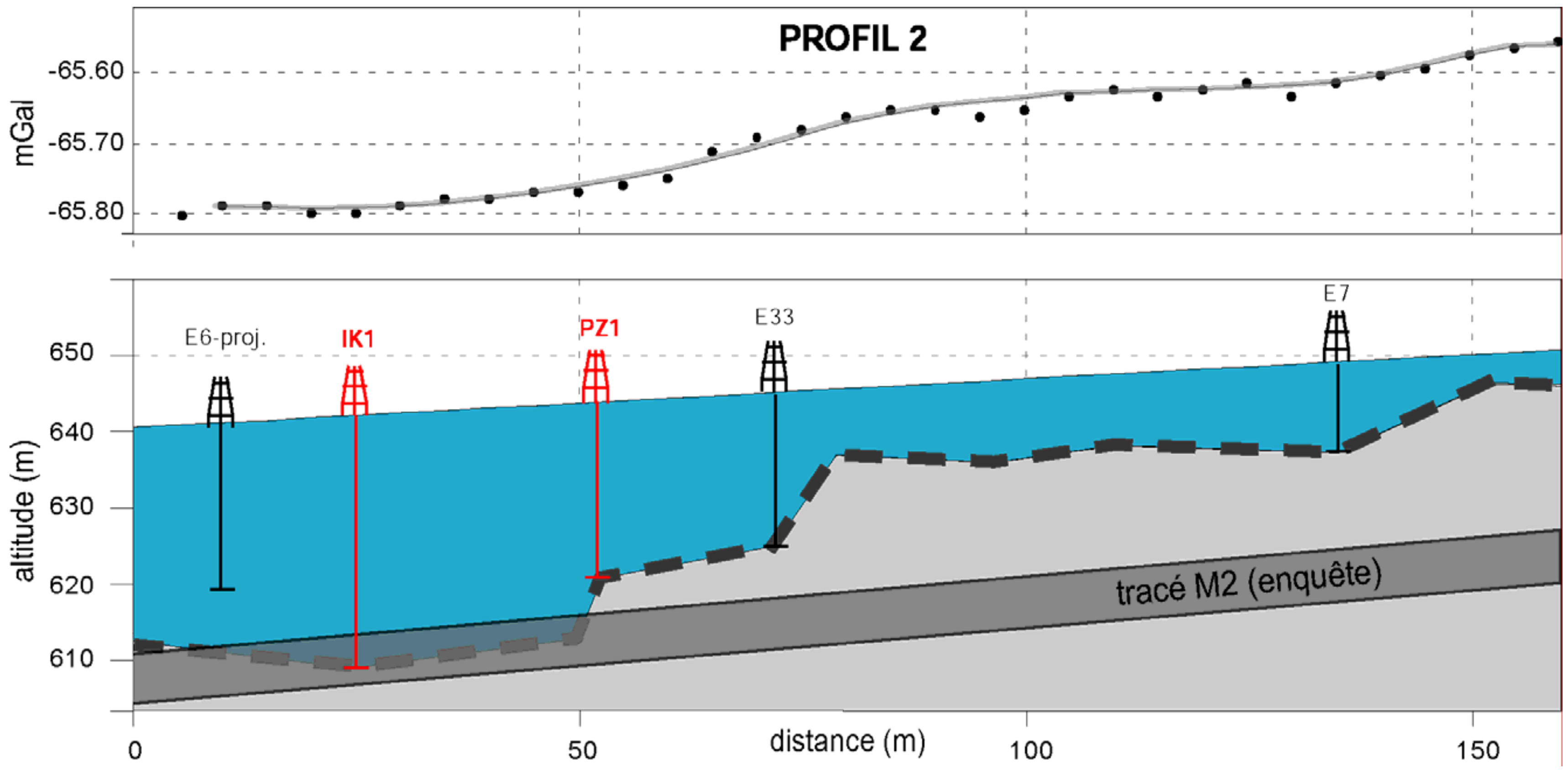
Source: P. Radogna et al.

Residual Anomaly



Source: P. Radogna et al.

Result…



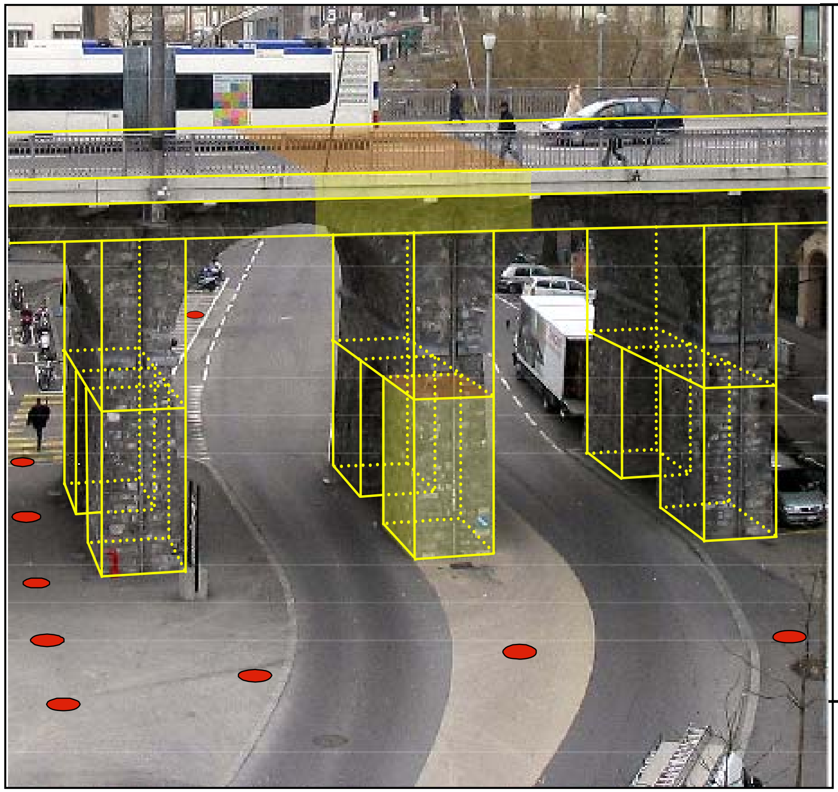
Source: P. Radogna et al.

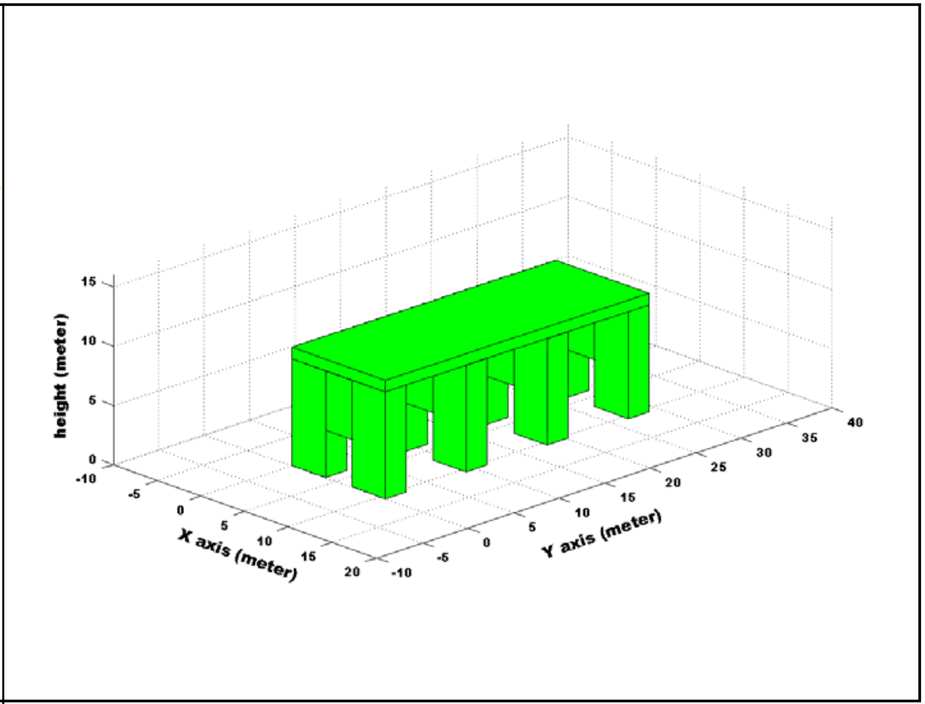
Complex building corrections

Painting of the valley and the bridge before 1874
and actual picture of the same zone

Source: P. Radogna et al.

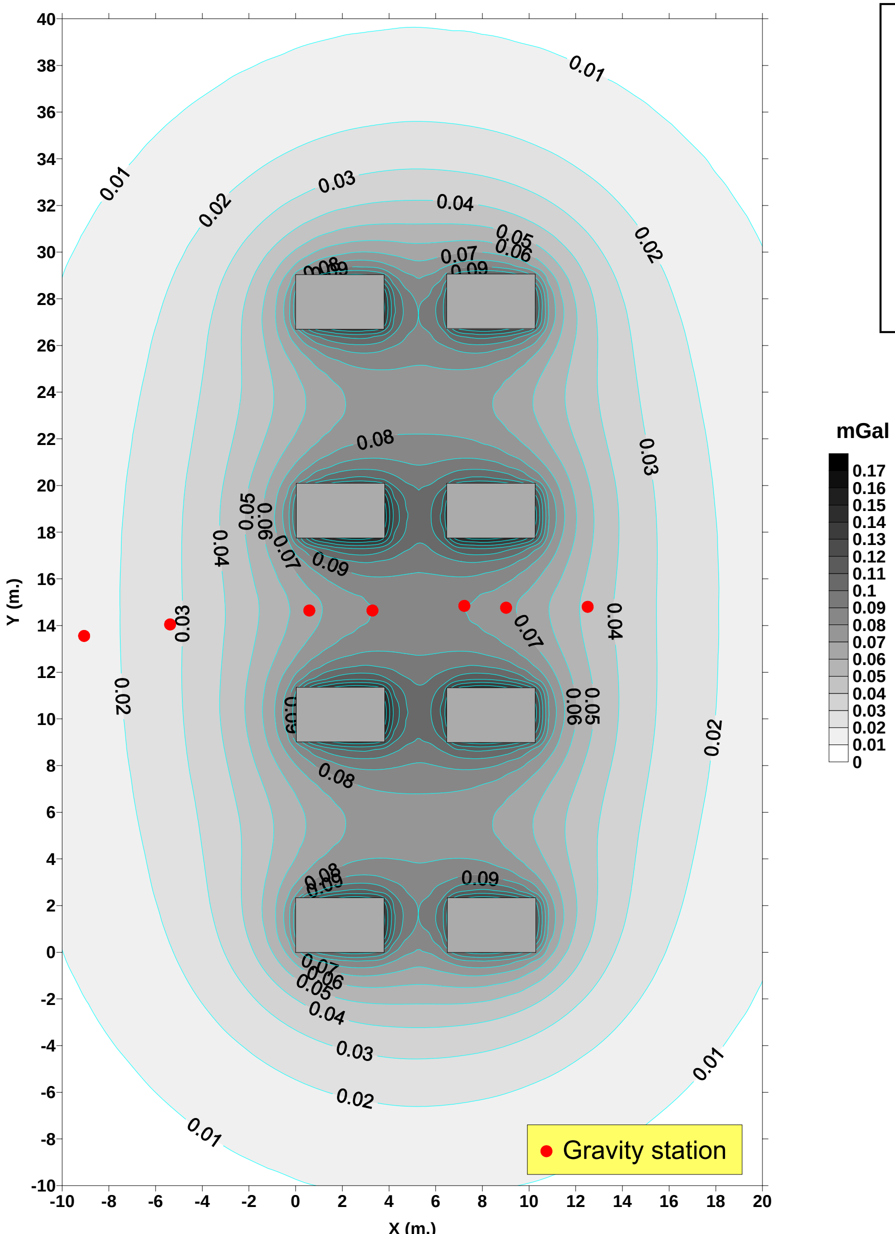
Rectangular prisms are used for modeling the bridge's pillars





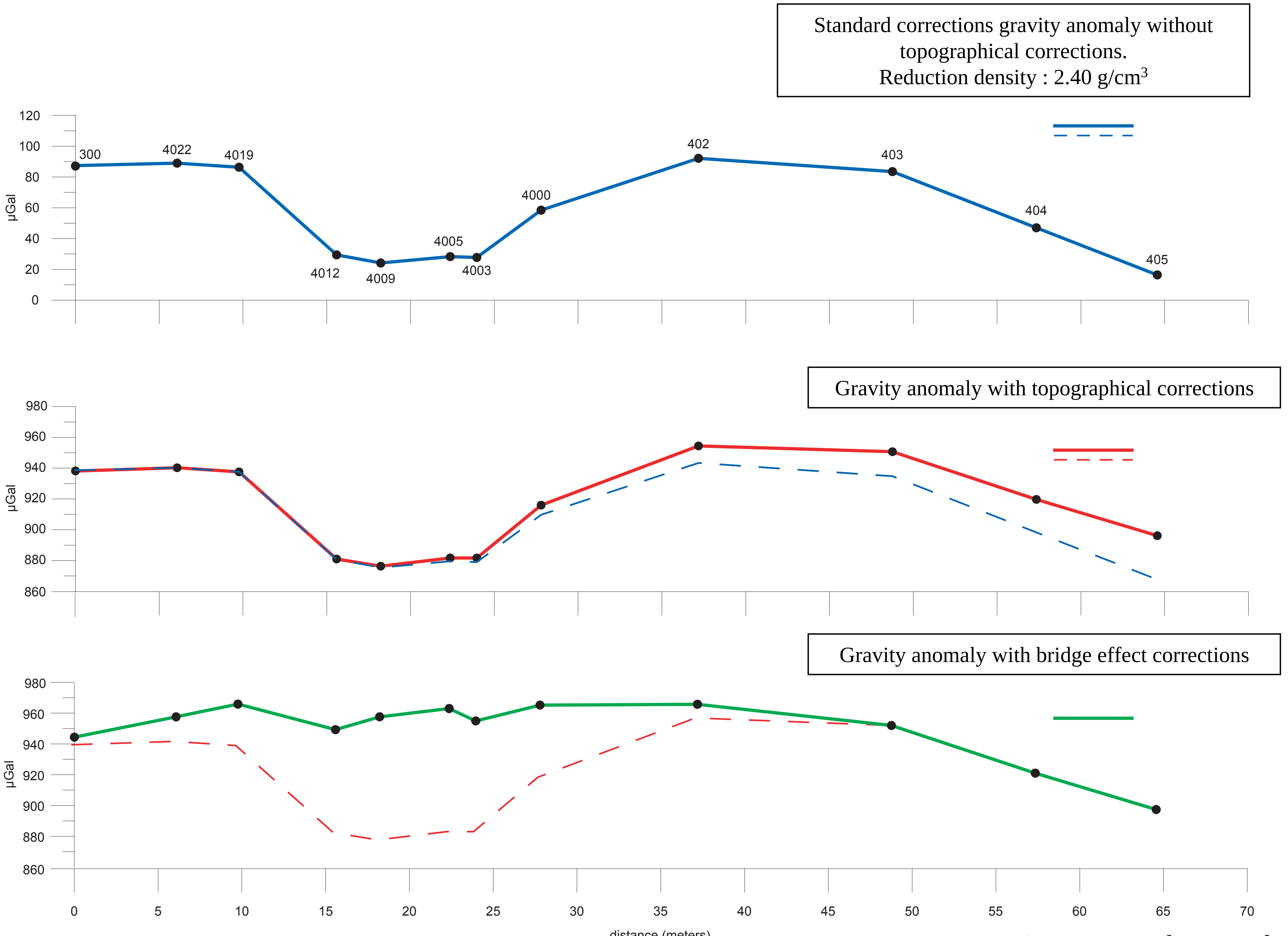
Source: P. Radogna et al.

Gravity effect of the bridge



- Formulation of rectangular prism (Nagy, 1966)
- Pillar's density is fixed to 2.00 g/cm^3

Source: P. Radogna et al.



Source: P. Radogna et al.

6. Conclusions

Advantages

- The only geophysical method that describes directly the density of the subsurface materials
- No artificial source required
- Useful in urban environment!

Drawbacks

- Expensive
- Complex acquisition process
- Complex data processing
- Limited resolution
- Very sensitive to non-unicity in the modeling solutions